Vibrational Coherences in Single Electron Tunneling through Nanoscale Oscillators

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21.03.2008

Outline

Motivation

- Experiments with free standing nanostructures

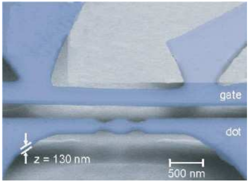


Figure: E. M. Weig *et al.*, PRL **92** 046804, (2004)

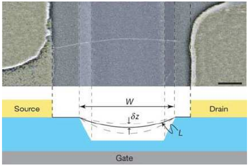


Figure: V. Sazanova *et al.*, Nature **431** 284, (2004)

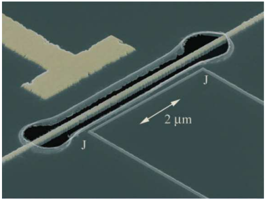


Figure: A. Naik *et al.*, Nature **443** 193, (2006)

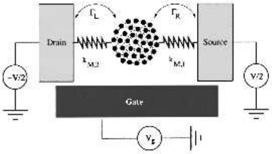
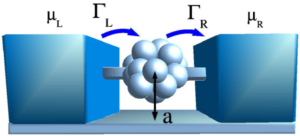


Figure: H. Park *et al.*, Nature **407** 57, (2000)

Model

Single electron transport through a vibrating Quantum Dot



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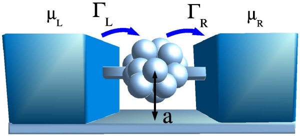

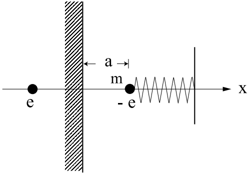

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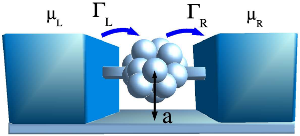
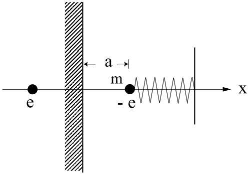




$$H = \frac{\mathbf{P}^2}{2m} + \frac{1}{2} m {\omega_0}^2 (\mathbf{a} - \mathbf{r})^2 - d^\dagger d \frac{e^2}{4\pi\epsilon_0} \frac{1}{2\mathbf{r} \cdot \mathbf{a}} \tag{1}$$

dot creation and annihilation operators;

$$d^\dagger = |1\rangle_{el} \langle 0|_{el} \qquad d = |0\rangle_{el} \langle 1|_{el}. \tag{2}$$

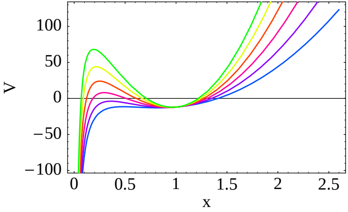
Eigenstates $|n, \nu\rangle = |n\rangle_{vib} \otimes |\nu\rangle_{el}$

Superposition of Hydrogen atom and harmonic oscillator

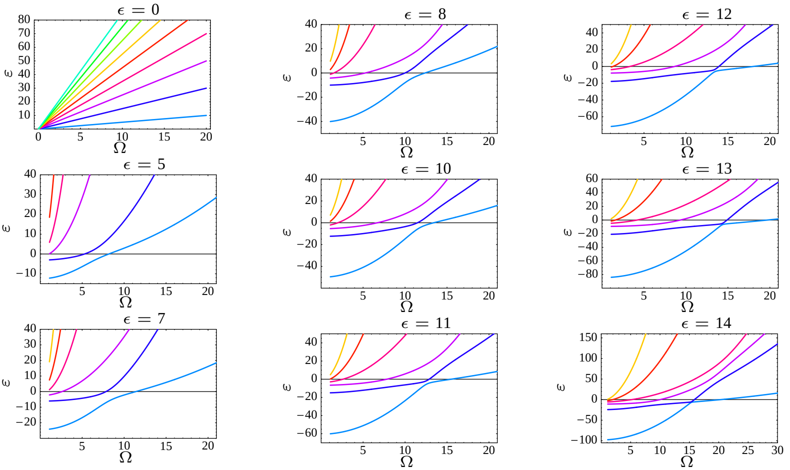
$$H = -\frac{1}{2}\frac{\partial^2}{\partial x^2} + \frac{\Omega^2}{2}(1-x)^2 - \frac{\epsilon}{x} \tag{3}$$

where

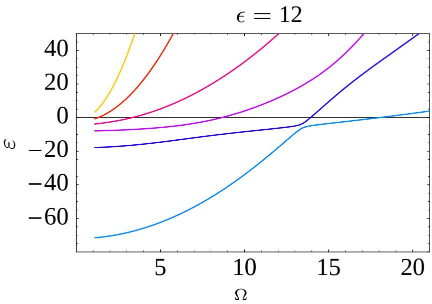
$$\frac{x}{a} \to x \quad , \qquad \frac{1}{\hbar^2} m^2 {\omega_0}^2 a^4 \to \Omega^2 \quad , \qquad \frac{ame^2}{8\hbar^2\pi\epsilon_0} \to \epsilon \tag{4}$$



Spectrum



- Consider $\epsilon = 12$



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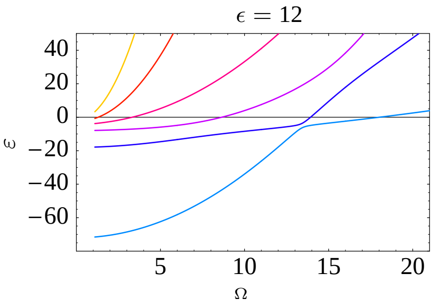
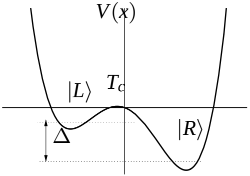


- Effective Two-Level-System at $\Omega \approx 13$

Why two level system?

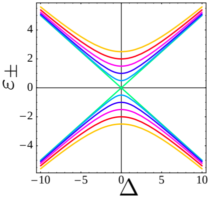
$$H = \begin{pmatrix} \frac{\Delta}{2} & T_c \\ T_c & -\frac{\Delta}{2} \end{pmatrix}$$



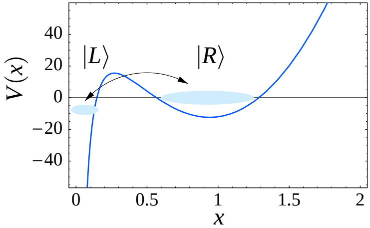
Eigensystem:

$$\varepsilon_{\pm} = \pm\frac{1}{2}\sqrt{\Delta^2 + 4T_c^2} \qquad (5)$$

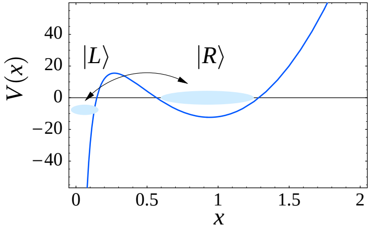
$$|\pm\rangle = \alpha\,|L\rangle \pm \beta\,|R\rangle \qquad (6)$$



→ coherent tunneling of vibrational states

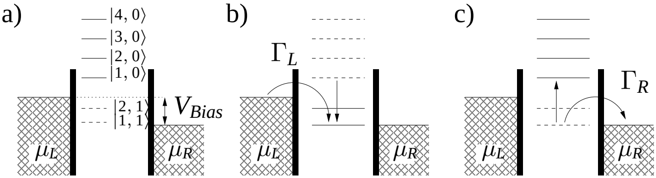


$\rightarrow$ coherent tunneling of vibrational states



How can we measure this?

Transport



$$H = H_{\text{Dot}} + H_{\text{Res}} + H_{\text{T}} \tag{7}$$

Eigenstates of H_{Dot} are denoted as $|n,\nu\rangle = |n\rangle_{vib} \otimes |\nu\rangle_{el}$.

$$H_{\text{Dot}} = \sum_{i,j} \varepsilon_{1,i} |i,1\rangle \langle i,1| + \varepsilon_{0,j} |j,0\rangle \langle j,0| \tag{8}$$

$$H_{\text{Res}} = \sum_{k_{L,R}} \varepsilon_k c_k^\dagger c_k \tag{9}$$

$$H_{\text{T}} = \sum_{k_{L,R}} T_k (c_k^\dagger d + d^\dagger c_k) \tag{10}$$

Master equation

Liouville-von Neumann equation in interaction picture

$$\dot{\tilde{\chi}}(t) = -i[\tilde{H}_{\mathrm{T}}(t), \tilde{\chi}(t)]. \tag{11}$$

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Tracing out the reservoirs and expanding the commutators yields to second order

$$\begin{aligned}
\dot{\tilde{\rho}}(t) = &- \sum_{k_{L,R}} \int_0^t dt' |T_k|^2 f(\varepsilon_k) e^{i\varepsilon_k(t-t')} [\tilde{d}(t)\tilde{d}^\dagger(t')\tilde{\rho}(t') - \tilde{d}^\dagger(t')\tilde{\rho}(t')\tilde{d}(t)] \\
&- \sum_{k_{L,R}} \int_0^t dt' |T_k|^2 (1 - f(\varepsilon_k)) e^{-i\varepsilon_k(t-t')} [\tilde{d}^\dagger(t)\tilde{d}(t')\tilde{\rho}(t') - \tilde{d}(t')\tilde{\rho}(t')\tilde{d}^\dagger(t)] \\
&- \sum_{k_{L,R}} \int_0^t dt' |T_k|^2 f(\varepsilon_k) e^{-i\varepsilon_k(t-t')} [\tilde{\rho}(t')\tilde{d}(t')\tilde{d}^\dagger(t) - \tilde{d}^\dagger(t)\tilde{\rho}(t')\tilde{d}(t')] \\
&- \sum_{k_{L,R}} \int_0^t dt' |T_k|^2 (1 - f(\varepsilon_k)) e^{i\varepsilon_k(t-t')} [\tilde{\rho}(t')\tilde{d}^\dagger(t')\tilde{d}(t) - \tilde{d}(t)\tilde{\rho}(t')\tilde{d}^\dagger(t')]
\end{aligned} \tag{12}$$

Matrix elements, Markov approximation ($\tilde{\rho}(t') \to \tilde{\rho}(t)$) and transform into Schrödinger picture:

$$\begin{aligned}
\frac{d}{dt}\langle n,\nu|\,\rho(t)\,|m,\mu\rangle = &- i(\varepsilon_{\nu n}-\varepsilon_{\mu m})\,\langle n,\nu|\,\rho_t\,|m,\mu\rangle - \\
&- \sum_{\alpha=L,R}\sum_{i,j,i',j'}\{\gamma_\alpha(\varepsilon_{1i}-\varepsilon_{0j'})\tau_{in}\tau_{ij'}\,\langle j',0|\,\rho_t\,|m,\mu\rangle\,\delta_{\nu 0} \\
&\qquad - \gamma_\alpha(\varepsilon_{1n}-\varepsilon_{0j})\tau_{nj}\tau_{mj'}\,\langle j,0|\,\rho_t\,|j',0\rangle\,\delta_{\nu 1}\delta_{\mu 1}+ \\
&\qquad + \overline{\gamma}_\alpha(\varepsilon_{1i'}-\varepsilon_{0j})\tau_{nj}\tau_{i'j}\,\langle i',1|\,\rho_t\,|m,\mu\rangle\,\delta_{\nu 1}- \\
&\qquad - \overline{\gamma}_\alpha(\varepsilon_{1i}-\varepsilon_{0n})\tau_{in}\tau_{i'm}\,\langle i,1|\,\rho_t\,|i',1\rangle\,\delta_{\nu 0}\delta_{\mu 0}+ \\
&\qquad + \gamma_\alpha(\varepsilon_{1i'}-\varepsilon_{0j})\tau_{i'j}\tau_{i'm}\,\langle n,\nu|\,\rho_t\,|j,0\rangle\,\delta_{\mu 0}- \\
&\qquad - \gamma_\alpha(\varepsilon_{1m}-\varepsilon_{0j'})\tau_{nj}\tau_{mj'}\,\langle j,0|\,\rho_t\,|j',0\rangle\,\delta_{\nu 1}\delta_{\mu 1}+ \\
&\qquad + \overline{\gamma}_\alpha(\varepsilon_{1i}-\varepsilon_{0j'})\tau_{ij'}\tau_{mj'}\,\langle n,\nu|\,\rho_t\,|i,1\rangle\,\delta_{\mu 1}- \\
&\qquad - \overline{\gamma}_\alpha(\varepsilon_{1i'}-\varepsilon_{0m})\tau_{in}\tau_{i'm}\,\langle i,1|\,\rho_t\,|i',1\rangle\,\delta_{\nu 0}\delta_{\mu 0}\}
\end{aligned} \tag{13}$$

Franck-Condon principle

The τ_{ij} are the Franck-Condon factors

$$\tau_{ij} = {}_1\langle i|j\rangle_0 = \int_0^\infty \psi_{1,i}(x)\psi_{0,j}(x) \tag{14}$$

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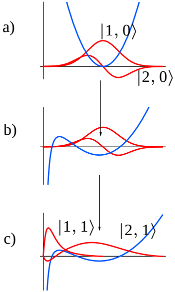

Current

Rates $\gamma_\alpha(\varepsilon) = \Gamma_\alpha f_\alpha(\varepsilon)$ for tunnelling onto the dot and $\overline{\gamma}_\alpha(\varepsilon) = \Gamma_\alpha(1 - f_\alpha(\varepsilon))$ for tunnelling off the dot.

$$\langle I \rangle = -\sum_{ij} e(\gamma^L(\varepsilon_{1i} - \varepsilon_{0j})\rho_{i_1 i_1} - \gamma^L(\varepsilon_{1i} - \varepsilon_{0j})\rho_{j_0 j_0}) \qquad (15)$$

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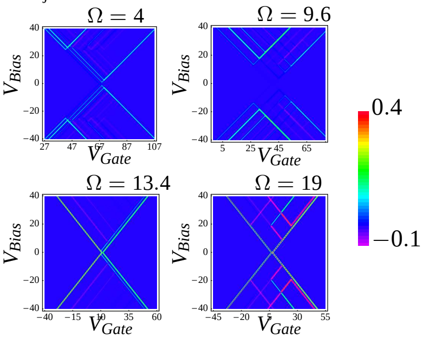

→ Franck-Condon blockade (nice), but where is the oscillation?

Shot Noise

- Current as a series of single electron transistions

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- Defined from current-current correlation

$$S(\omega) = \int_{-\infty}^{\infty} d\tau e^{i\omega\tau} \langle [\Delta I(t), \Delta I(t+\tau)]_{+} \rangle \tag{16}$$

where $\Delta I(t) = I(t) - \langle I \rangle$

Superoperators

- Master equation as a Liouvillian superoperator

$$\dot{\rho}(t) = \mathcal{L}\rho(t) = (\mathcal{L}_0 + \mathcal{L}_J)\rho(t) \tag{17}$$

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$$S(\omega) = \langle\langle 0| \mathcal{L}_J |0\rangle\rangle - 2\mathrm{Re}[\langle\langle 0| \mathcal{L}_J \mathcal{R}(\omega) \mathcal{L}_J |0\rangle\rangle] \tag{19}$$

where $\mathcal{R}(\omega)$ is the pseudo inverse of $(i\omega - \mathcal{L})$

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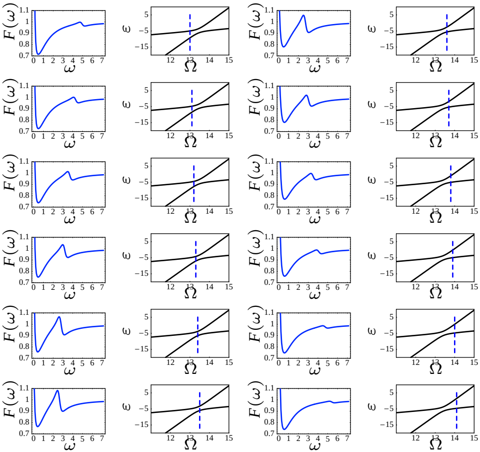
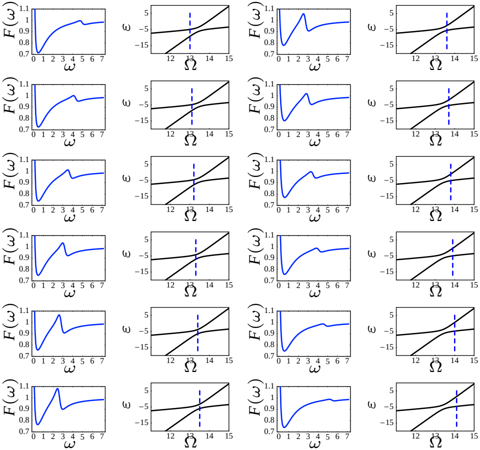
- Fano factor

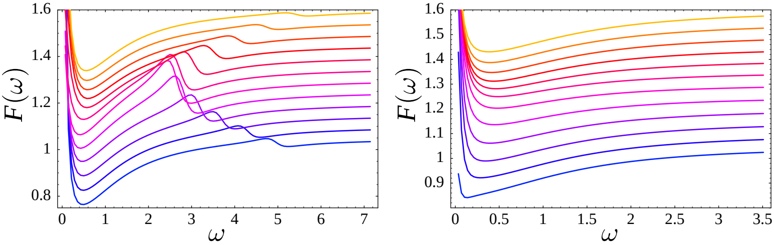
$$F(\omega) = \frac{S(\omega)}{\langle I \rangle} \tag{20}$$


F(ω)
ω
Ω



→ coherent tunneling of the vibrational state.

Comparison with a rate equation approach using Fermis Golden Rule:



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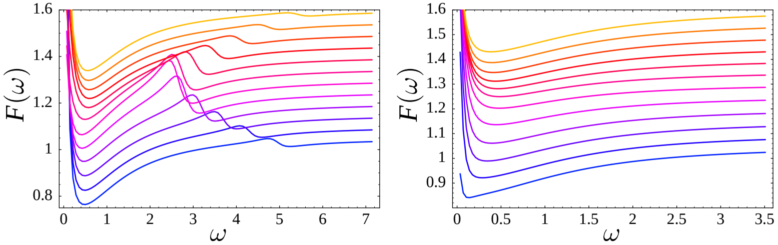

$\longrightarrow$ Keeping the coherences is crucial for this feature.

Conlusion

- Model for single electron transport through vibrating qunatum dot with Image charge
- Two level system points to coherent oscillations
- Franck-Condon Master equation
- Coherent interaction visible in noise spectrum due to cohrences in master equation

Thank you very much!!

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Lets go to lunch!!