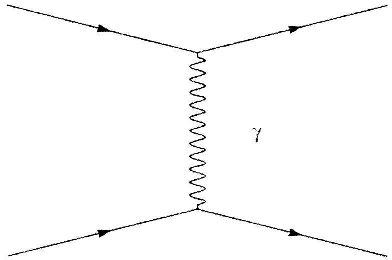
Introduction to Feynman Diagrams



Igor Reshetnyak

Before we start...

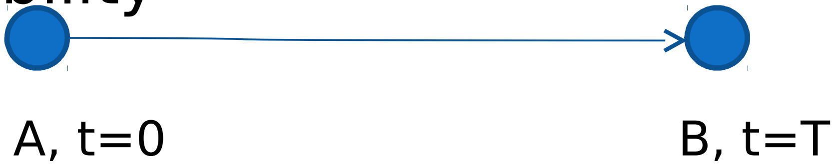
- Friday 13th
- Programmers’ day (256th day of the year)
  - Some numerical applications
  - Less derivations, more discussions

Plan

- Classical dynamics
  - Trajectories
  - Least action principle
- Quantum mechanics (Feynmann formulation)
  - Functional integral
  - Applications
- Quantum field theory & diagrams
  - What changes?
  - Feynmann diagrams

Classical Dynamics

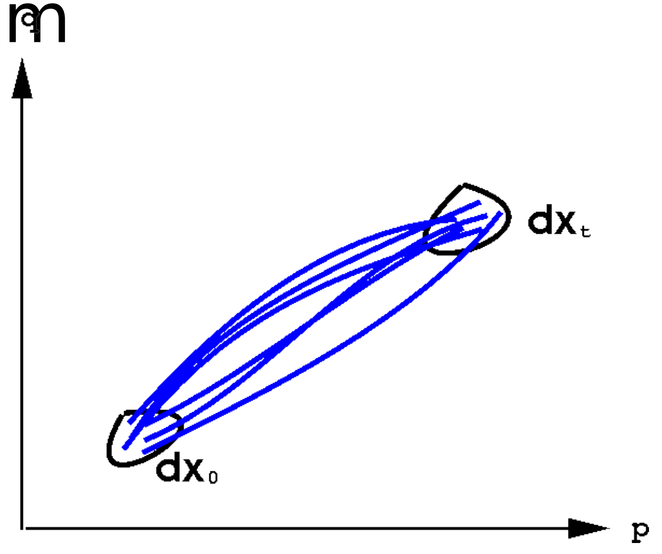
- Deterministic picture
- One can track the full evolution
- Non-trivial things:
  - Initial conditions
  - Instability



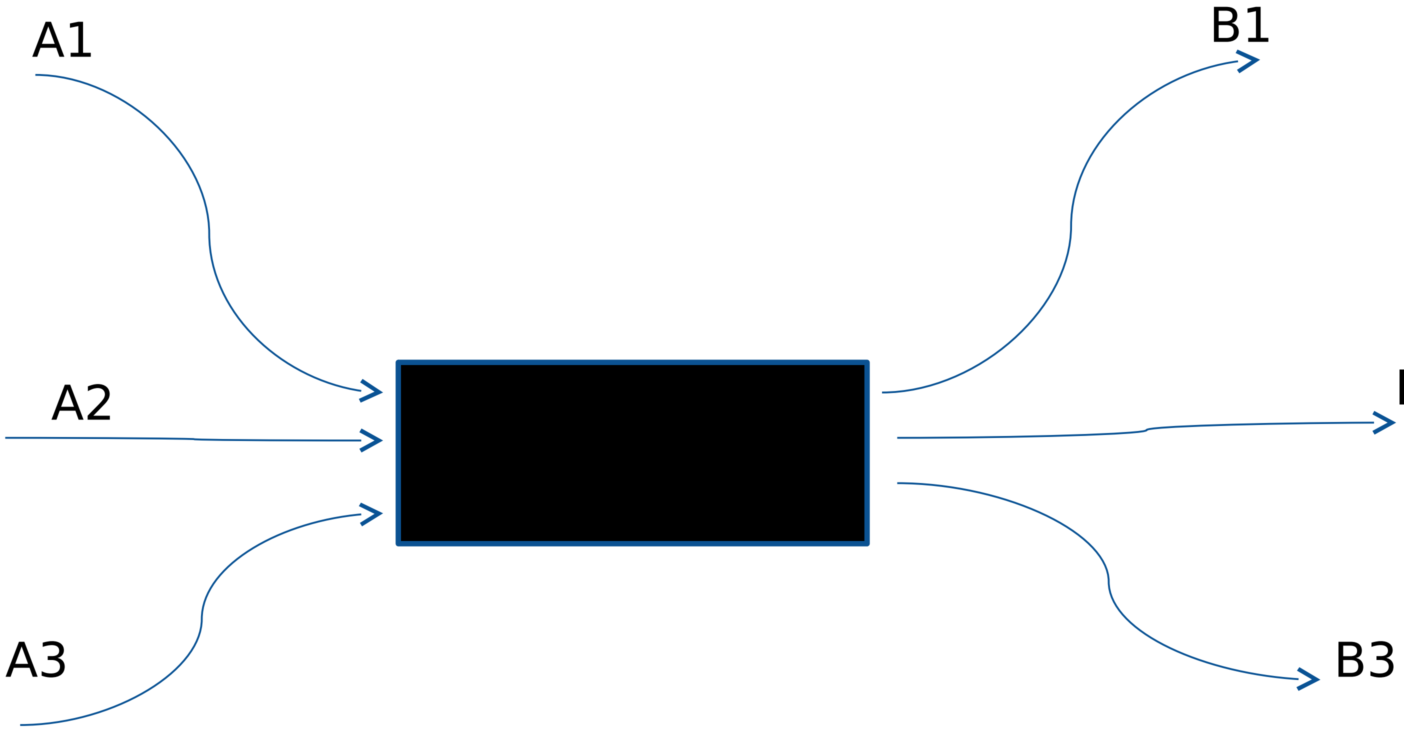
Classical Dynamics

Trajectory - path taken by the system from point A to point B in phase space (3N)

Liouville Theorem



Classical Dynamics



Inverse scattering problem!

Least action principle

- One trajectory
- We know the initial and final state of the system

$$S[\mathbf{q}(t)] = \int_{t_1}^{t_2} L(\mathbf{q}(t), \dot{\mathbf{q}}(t), t)dt$$

$$\delta S = 0$$

From optics... Ferma's principle:
Light takes the shortest path

$$\frac{\partial L}{\partial \mathbf{q}} - \frac{d}{dt}\frac{\partial L}{\partial \dot{\mathbf{q}}} = 0$$

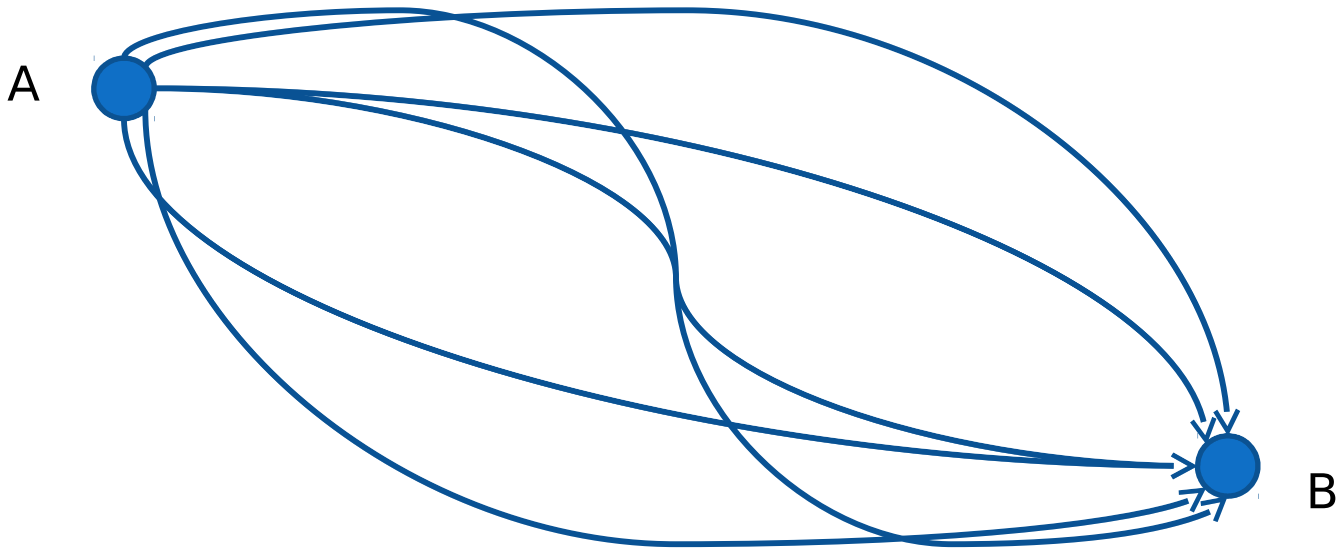
Classical Statistical Physics

- Paths still deterministic
- Initial conditions (Distribution)
- Nothing really new…

Quantum Physics

- **Quantum physics is a theory of transformation**
  - We CANNOT track full the path of the system
  - Continuous measurement ⟹ quantum Zeno effect

Quantum Physics



Multiple trajectories!

Quantum Mechanics

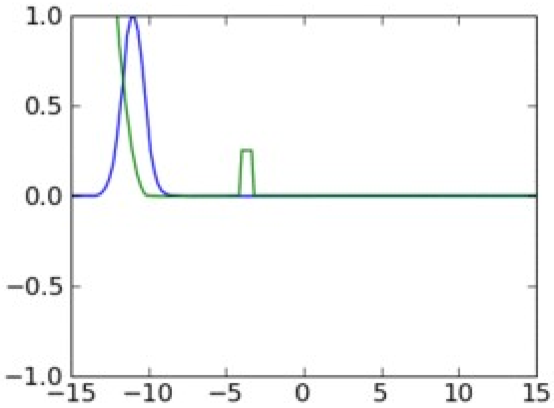
- Multiple trajectories
- Weight ~ exp(i S)
- **Not always the classical S** e.g. L = f(q) $(dq/dt)^2$

$$\psi_t(y) = \int \psi_0(x) K(x-y;t)\,dx = \int \psi_0(x) \int_{x(0)=x}^{x(t)=y} e^{iS} Dx$$

Application 1

Do things actually work?

$$\exp\left[-i\,\Delta t\,(\frac{p^2}{2}+V(x))\right] \simeq \exp\left[-iV(x)\,\Delta t/2\right]\,\exp\left[-i\,\Delta t\frac{p^2}{2}\right]\,\exp\left[-iV(x)\,\Delta t\,/2\right]$$



http://cours-physique.lps.ens.fr/index.php/TD5_The_Evolution_Operator_2012

Quantum Statistics

- Rather similar

$$Z = \sum_j \langle j | e^{-\beta H} | j \rangle = \mathrm{Tr} e^{-\beta H}.$$

- Changes:
  - Imaginary time
  - Periodical
  - L[q,i dq/dt]

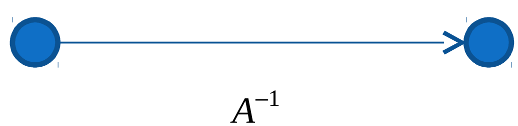
Field Theory

- Fields i.e. functions of x and t
- Identical particles…
- But in the end we still have our functional integral…

Gaussian integrals

$$\int_{-\infty}^{\infty} \exp\left(-\frac{1}{2}\sum_{i,j=1}^{n} A_{ij} x_i x_j\right) d^n x = \int_{-\infty}^{\infty} \exp\left(-\frac{1}{2} x^T A x\right) d^n x = \sqrt{\frac{(2\pi)^n}{\det A}}$$

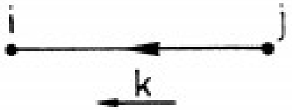
$$\int x^{k_1} \cdots x^{k_{2N}} \exp\left(-\frac{1}{2}\sum_{i,j=1}^{n} A_{ij} x_i x_j\right) d^n x = \sqrt{\frac{(2\pi)^n}{\det A}} \frac{1}{2^N N!} \sum_{\sigma \in S_{2N}} (A^{-1})^{k_{\sigma(1)} k_{\sigma(2)}} \cdots (A^{-1})^{k_{\sigma(2N-1)} k_{\sigma(2N)}}$$



Feynman rules

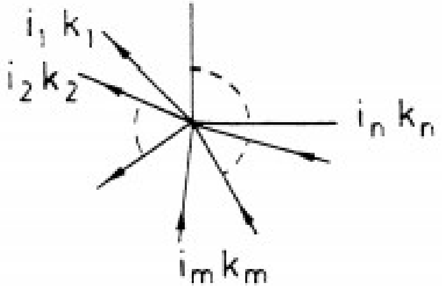
(Almost the same thing)

$$\mathcal{L}(x) = \psi_i^*(x) V_{ij} \psi_j(x) + \frac{1}{2} \phi_i(x) W_{ij} \phi_j(x) + \mathcal{L}_I(\psi^*, \psi, \phi)$$



$$\Delta_{Fij}(k) = -\frac{1}{(2\pi)^4 i}\left[\bar{V}^{-1}(k)\right]_{ij} ,$$

$$\Delta_{Fij}(k) = -\frac{1}{(2\pi)^4 i}\left[\frac{1}{2}\bar{W}(k) + \frac{1}{2}\tilde{\bar{W}}(-k)\right]^{-1}_{ij} .$$



$$(2\pi)^4 i \sum_{\{1\ldots m-1\}} \sum_{\{m\ldots n-1\}} \sum_{\{n\ldots\}} (-i)^p \times$$

$$\times\, \alpha_{i_1\ldots}(k, k_1, k_2, \ldots)\, \delta_4(k + k_1 + \ldots)$$

Thank you for your attention!

Bibliography

- Main texts:
  - Weinberg "Quantum Theory of Fields" (particularly vol 1 chapter 9)
  - Abrikosov, Gor'kov, Dzjaloshinskii “Methods of quantum field theory in condensed matter physics”
  - Feynmann, Hibs "Quantum mechanics and path integrals“
- Philosophical things:
  - (Russian) http://mezhpr.fizteh.ru/biblio/q-ivanov/quant-1-0-arphanncajc.pdf
- Texts online:
  - http://arxiv.org/pdf/quant-ph/0004090v1.pdf
  - http://www.pa.msu.edu/~pratts/phy831/lectures/lectures.pdf
  - http://cds.cern.ch/record/186259/files/CERN-73-09.pdf?version=1
  - http://web.mit.edu/dikaiser/www/FdsAmSci.pdf
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