

Energy Loss Beamline

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European Theoretical Spectroscopy Facility (ETSF)

22 October 2010

Outline

Information about the Beamline

Calculation(microscopic)-Measurement(macroscopic) Connection

Underlying theory and approximations

Usefulness of Theory

Codes

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Calculation(microscopic)-Measurement(macroscopic) Connection

Underlying theory and approximations

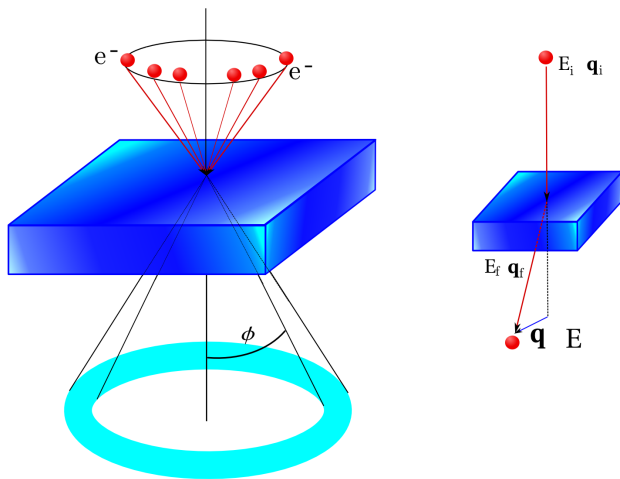
Usefulness of Theory

Codes

The Energy Loss Beamline

- Coordinator: Francesco Sottile
(Palaiseau, Francesco.Sottile@polytechnique.fr)
- Experiments:
 - Electron Energy Loss Spectroscopy (EELS).
 - Reflection Electron Energy Loss Spectroscopy (REELS).
 - Inelastic and Coherent X-ray Scattering (IXS,CIXS).
- Systems:
 - Bulk, Surfaces and Nanostructures.
- Methods:
 - Density functional approach: DFT-KS, TDDFT.
 - Many-body techniques: Bethe-Salpeter Equation.
- Nodes involved:
 - Palaiseau, Rome, Milan, Grenoble

Spectroscopy: Electron or X Scattering

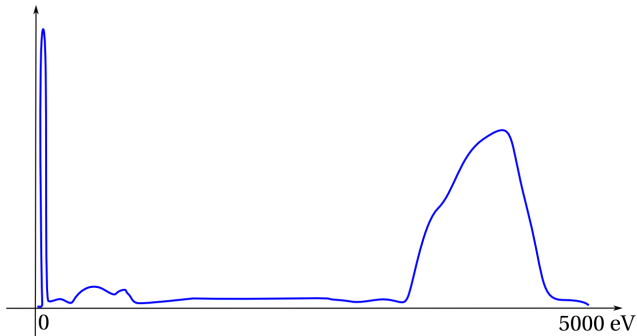


Spectroscopy: Electron or X Scattering

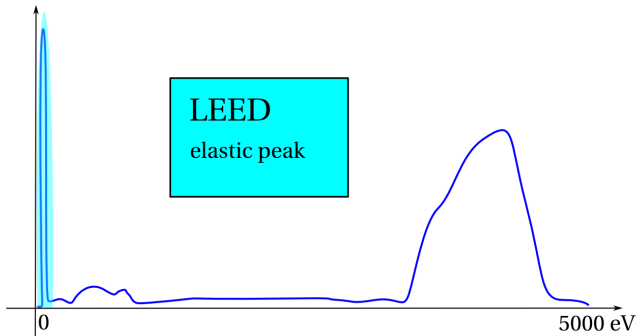
Energy Loss Function

$$\frac{d^2\sigma}{d\Omega dE} \propto \text{Im} \left\{ \epsilon^{-1} \right\}$$

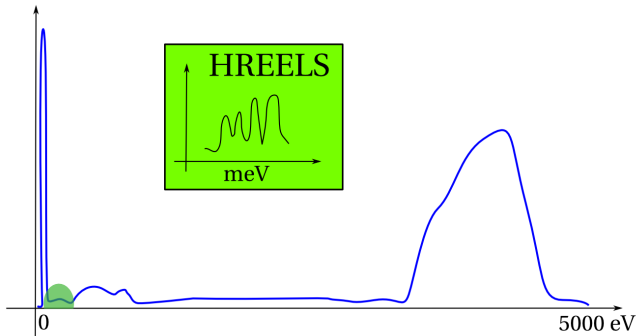
Spectroscopy: Electron or X Scattering



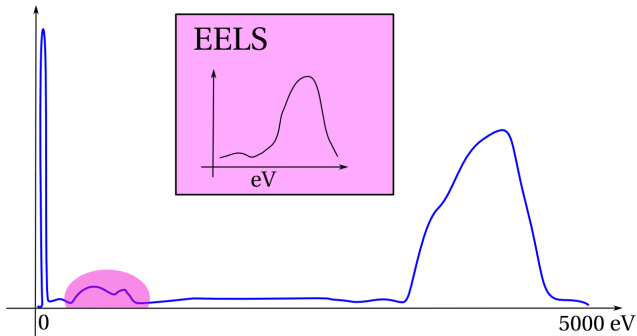
Spectroscopy: Electron or X Scattering



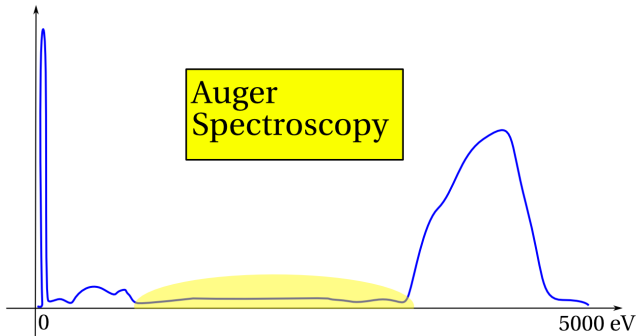
Spectroscopy: Electron or X Scattering



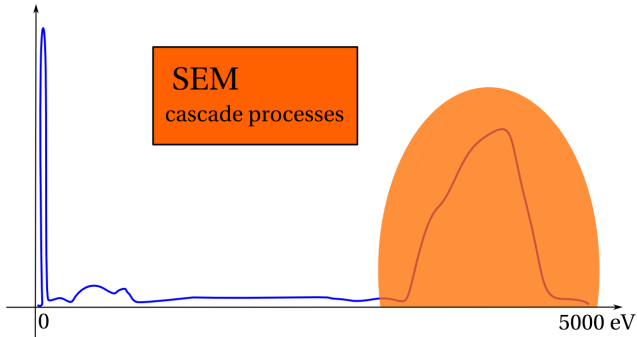
Spectroscopy: Electron or X Scattering



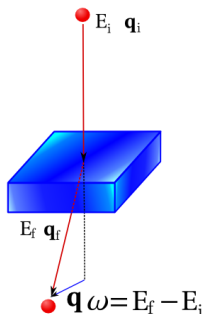
Spectroscopy: Electron or X Scattering



Spectroscopy: Electron or X Scattering



Spectroscopy: Electron or X Scattering



$$\frac{d^2\sigma}{d\Omega dE} \propto \text{Im} \{ \epsilon^{-1} \}$$

$$\epsilon_M^{-1}(\mathbf{q}, \omega)$$

frequency-momentum
dependent inverse
Macroscopic dielectric
function

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Microscopic-Macroscopic Connection

Theoretical definition

$$\mathbf{E}(\mathbf{r}, \omega) = \int d\mathbf{r}' \varepsilon^{-1}(\mathbf{r}, \mathbf{r}', \omega) \mathbf{D}(\mathbf{r}', \omega)$$

constitutive closure to Maxwell equations

The connection ?

$$\varepsilon^{-1}(\mathbf{r}, \mathbf{r}', \omega) \implies \varepsilon_{\text{M}}^{-1}(\mathbf{q}, \omega)$$

microscopic

macroscopic

Theoretical definition in G space

$$\mathbf{E}(\mathbf{q} + \mathbf{G}, \omega) = \varepsilon_{\mathbf{G}, \mathbf{G}'}^{-1}(\mathbf{q}, \omega) \mathbf{D}(\mathbf{q} + \mathbf{G}', \omega)$$

$$\begin{array}{c}
 G_0=0 \\
 G_1 \\
 G_2 \\
 G_3 \\
 \vdots
 \end{array}
 \left(\begin{array}{c}
 \text{---} \\
 \text{~} \\
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 \vdots
 \end{array} \right)
 =
 \left(\begin{array}{cccc}
 \mathcal{E}_{00}^{-1} & \mathcal{E}_{G_0 G_1}^{-1} & \mathcal{E}_{G_0 G_2}^{-1} & \mathcal{E}_{G_0 G_3}^{-1} & \cdots \\
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 \mathcal{E}_{G_2 G_0}^{-1} & \mathcal{E}_{G_2 G_1}^{-1} & \mathcal{E}_{G_2 G_2}^{-1} & \mathcal{E}_{G_2 G_3}^{-1} & \\
 \mathcal{E}_{G_3 G_0}^{-1} & \mathcal{E}_{G_3 G_1}^{-1} & \mathcal{E}_{G_3 G_2}^{-1} & \mathcal{E}_{G_3 G_3}^{-1} & \\
 \vdots & \vdots & & & \ddots
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Theoretical definition in G space

$$\mathbf{E}(\mathbf{q} + \mathbf{G}, \omega) = \varepsilon_{\mathbf{G},0}^{-1}(\mathbf{q}, \omega) \mathbf{D}(\mathbf{q} + 0, \omega)$$

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Ab initio approach to calculate ε

Full polarizability

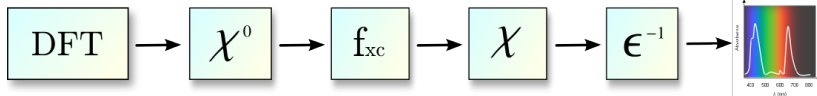
- TDDFT $:: \chi = \chi^0 + \chi^0 (v + f_{xc}) \chi$
- BSE $:: {}^4\chi = {}^4\chi^0 + {}^4\chi^0 ({}^4v + {}^4\xi) {}^4\chi$

Dielectric function

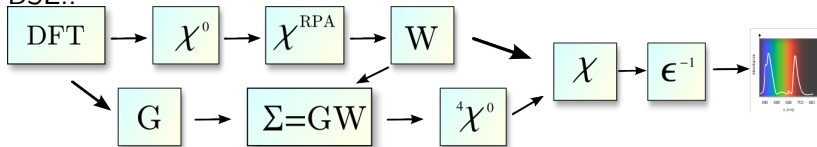
$$\varepsilon_{GG'}^{-1}(\mathbf{q}, \omega) = 1 + v_{\mathbf{G}}(\mathbf{q}) \chi_{\mathbf{G}, \mathbf{G}'}(\mathbf{q}, \omega)$$

Ab initio approach to calculate ε

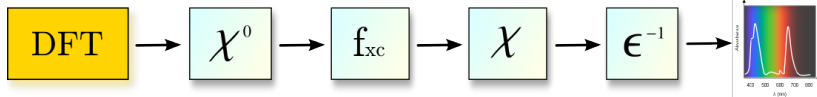
TDDFT::



BSE::



First step: the ground state calculation

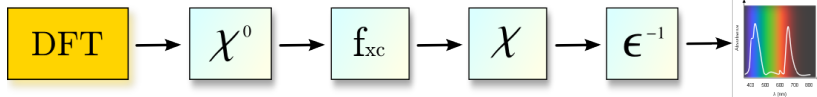


- DFT with plane waves basis $\psi(\mathbf{r}) = \sum_{\mathbf{G}} c_{\mathbf{G}} e^{i\mathbf{G} \cdot \mathbf{r}}$
- Cutoff energy $E_{\text{cutoff}} = \frac{|\mathbf{G}_{\text{max}}|^2}{2}$ as a unique convergence parameter
- pseudopotential (norm-conserving)
- LDA, GGA exchange-correlation potential

Results :: Eigenvalues (and eigenvectors)

$$\psi_{nk}, \epsilon_{nk}, f_{nk}$$

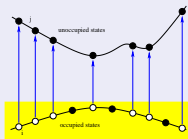
First step: the ground state calculation



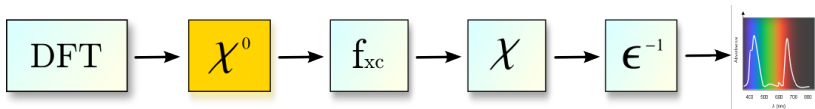
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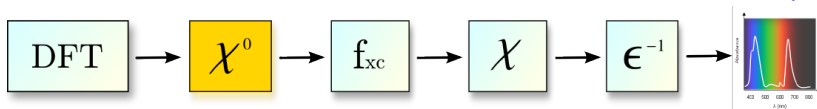
Second step: the Independent Particle Polarizability (IPA)



$$\chi^0(\mathbf{r}, \mathbf{r}', \omega) = \sum_{ij} \frac{\phi_i(\mathbf{r})\phi_j^*(\mathbf{r})\phi_i^*(\mathbf{r}')\phi_j(\mathbf{r}')}{\omega - (\epsilon_i - \epsilon_j)}$$

$$\chi_{\mathbf{G}, \mathbf{G}'}^0(\mathbf{q}, \omega) = \sum_{ij} \frac{\langle \phi_i | e^{i(\mathbf{q} + \mathbf{G})\mathbf{r}} | \phi_j \rangle \langle \phi_i | e^{-i(\mathbf{q} + \mathbf{G}')\mathbf{r}'} | \phi_j \rangle}{\omega - (\epsilon_i - \epsilon_j)}$$

Second step: the Independent Particle Polarizability (IPA)



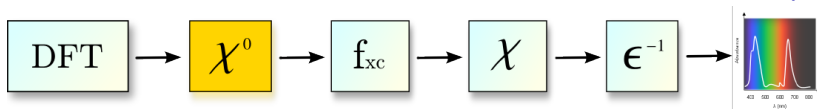
IPA :: the easy way

$$\varepsilon_{\mathbf{G}\mathbf{G}'}(\mathbf{q}, \omega) = 1 - v_{\mathbf{G}}(\mathbf{q})\chi_{\mathbf{G},\mathbf{G}'}^0(\mathbf{q}, \omega)$$

$$\varepsilon_{00}(\mathbf{q}, \omega) = 1 - v_0(\mathbf{q})\chi_{00}^0(\mathbf{q}, \omega)$$

$$\text{ELS} = \text{Im} \left\{ \frac{1}{\varepsilon_{00}(\mathbf{q}, \omega)} \right\}$$

Second step: the Independent Particle Polarizability (IPA)



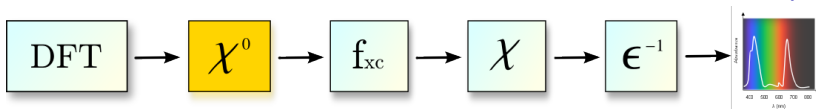
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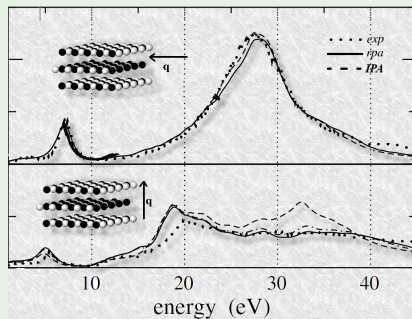
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Independent Particle Polarizability

Some good results ... (graphite)



A. Marinopoulos *et al.* Phys.Rev.Lett **89**, 76402 (2002)

ELS within RPA

RPA :: the inclusion of local fields

$$\varepsilon_{\mathbf{G}\mathbf{G}'}(\mathbf{q}, \omega) = 1 - v_{\mathbf{G}}(\mathbf{q})\chi_{\mathbf{G},\mathbf{G}'}^0(\mathbf{q}, \omega)$$

$$\varepsilon_{\mathbf{G}\mathbf{G}'}^{-1}(\mathbf{q}, \omega) \mapsto \varepsilon_{00}^{-1}(\mathbf{q}, \omega)$$

$$\text{ELS} = \text{Im} \{ \varepsilon_{00}^{-1}(\mathbf{q}, \omega) \}$$

ELS within RPA

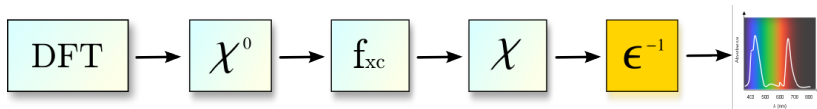
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ELS within RPA



RPA :: the inclusion of local fields

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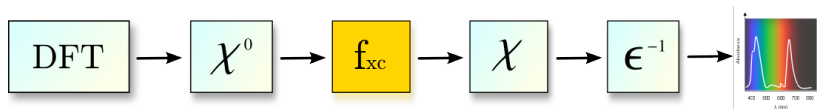
$$\text{ELS} = \text{Im} \left\{ \varepsilon_{00}^{-1}(\mathbf{q}, \omega) \right\}$$

Ab initio approach

Full polarizability :: RPA

- TDDFT :: $\chi = \chi^0 + \chi^0 (v + \cancel{\chi_c}) \chi$

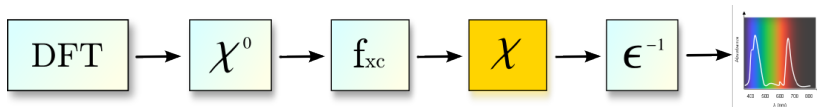
Beyond RPA :: through a kernel



ALDA kernel (GGA, EXX, etc.)

$$f_{xc} = \frac{\delta V_{xc}^{LDA}}{\delta n} \delta(\mathbf{r}, \mathbf{r}') (\omega = 0)$$

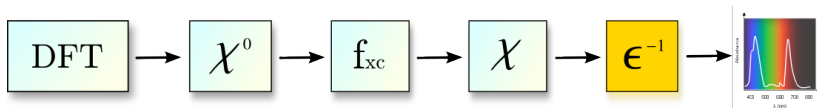
Beyond RPA :: through a kernel



Full polarizability :: ALDA

- TDDFT :: $\chi = \chi^0 + \chi^0 (v + f_{xc}^{ALDA}) \chi$

Beyond RPA :: through a kernel

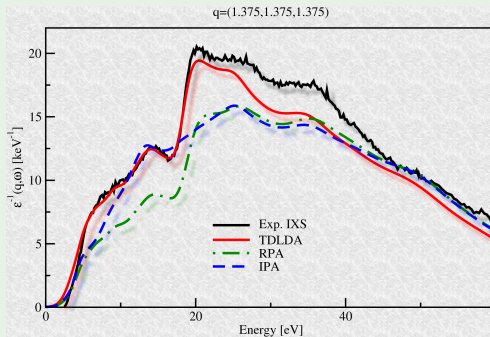


Dielectric function

$$\epsilon_{\mathbf{G}\mathbf{G}'}^{-1}(\mathbf{q}, \omega) = 1 + v_{\mathbf{G}}(\mathbf{q})\chi_{\mathbf{G},\mathbf{G}'}(\mathbf{q}, \omega)$$

ALDA results

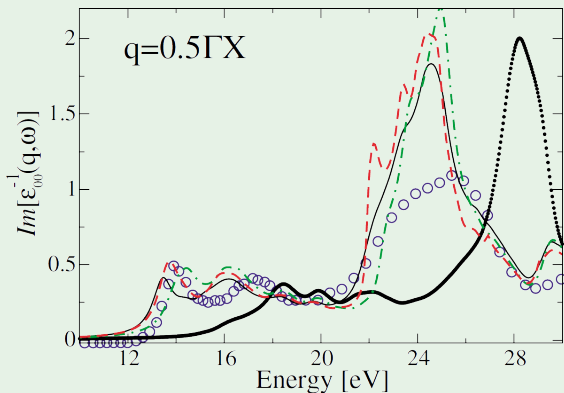
ALDA on IXS of Silicon



H-C. Weissker *et al.*, Physical Review Letters **97**, 237602 (2006)

BSE results

LiF - Energy Loss small non-zero momentum transfer



A.Marini, R. Del Sole and A.Rubio, *Physical Review Letters* **91**, 256402 (2003)

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Why the numerical approach is important

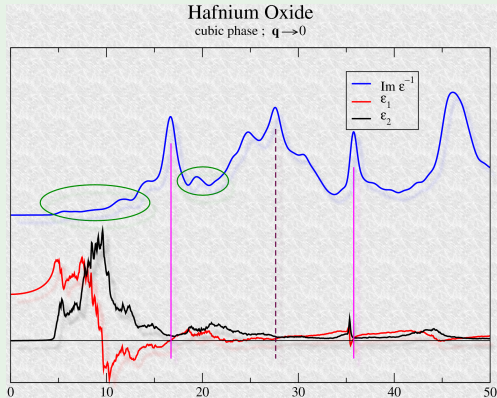
- Analysis
- Prediction

Why the numerical approach is important

- Analysis
- Prediction

Analysis

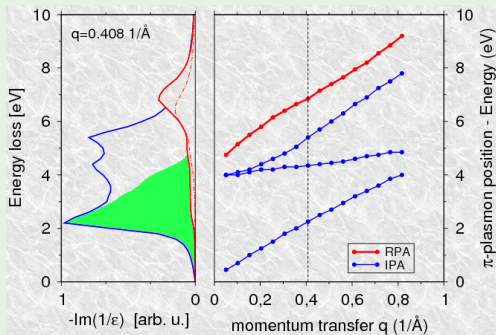
ELS of Hafnium Oxide



Zobelli and Sottile, to be published.

Analysis

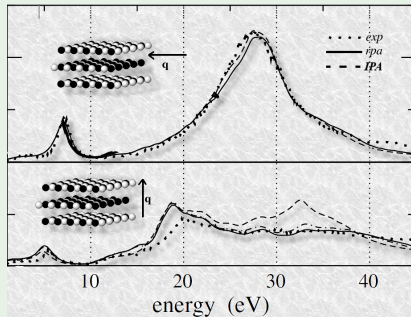
ELS of Nanotubes



Kramberger *et al.*, Phys. Rev. Lett. **100**, 196803 (2008)

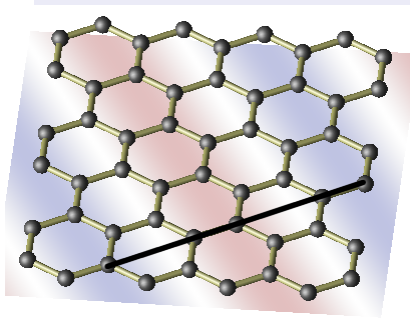
Analysis

Plasmons in graphite



Graphite: π -Plasmon

Induced charge fluctuations $\rho^{ind} = \chi\varphi^{ext}$



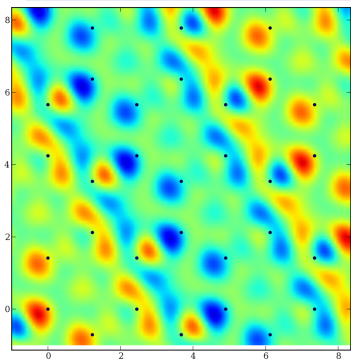
- plane wave perturbation

$$\varphi^{ext}(\mathbf{r}, \omega) \propto e^{-i(\omega t - \mathbf{q}\mathbf{r})}$$

- $|\mathbf{q}| = 0.74 \text{ \AA}^{-1}$, $\lambda = 8.5 \text{ \AA}$
- $\hbar\omega = 7\text{eV}$

Graphite: π -Plasmon

Induced charge fluctuations $\rho^{ind} = \chi\varphi^{ext}$



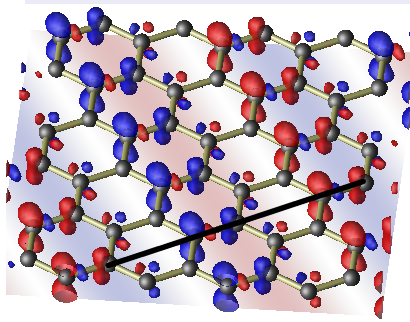
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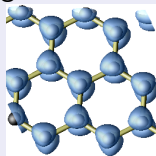
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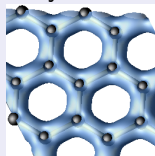
Induced charge fluctuations $\rho^{ind} = \chi\varphi^{ext}$



ground state density



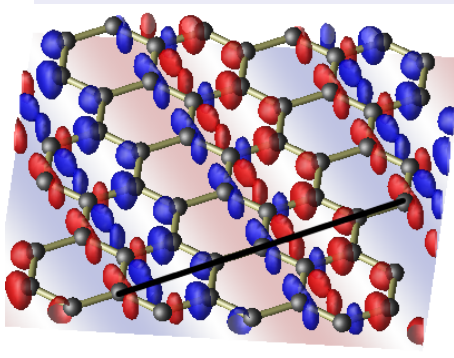
p_z - orbitals



sp^2 - orbitals

Graphite: $\pi + \sigma$ -Plasmon

Induced charge fluctuations $\rho^{ind} = \chi\varphi^{ext}$



- plane wave perturbation

$$\varphi^{ext}(\mathbf{r}, \omega) \propto e^{-i(\omega t - \mathbf{q}\mathbf{r})}$$

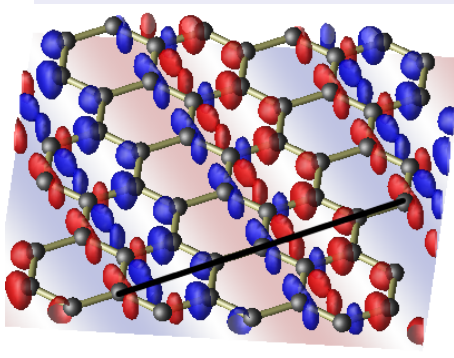
- $|\mathbf{q}| = 0.74 \text{ \AA}^{-1}$, $\lambda = 8.5 \text{ \AA}$
- $\hbar\omega = 30\text{eV}$



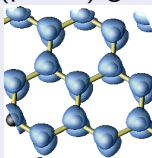
R. Hambach *et al.*, to be published.

Graphite: $\pi + \sigma$ -Plasmon

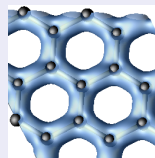
Induced charge fluctuations $\rho^{ind} = \chi\varphi^{ext}$



(partial) ground state density



p_z - orbitals



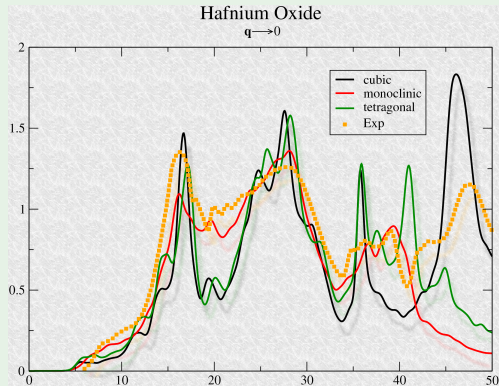
sp^2 - orbitals



R. Hambach *et al.*, to be published.

Prediction

ELS of Hafnium Oxide



Zobelli and Sottile, to be published.

Valence Electron Loss Spectroscopy

Theoretical support and analysis

Technical aspects

- Easy convergence
- Spectra in absolute value
- whole dielectric 2-rank tensor
- CIXS, circular dichroism with EELS

Limits

- 100-1000 atoms
- 0-100 eV

Valence Electron Loss Spectroscopy

Theoretical support and analysis

Analysis

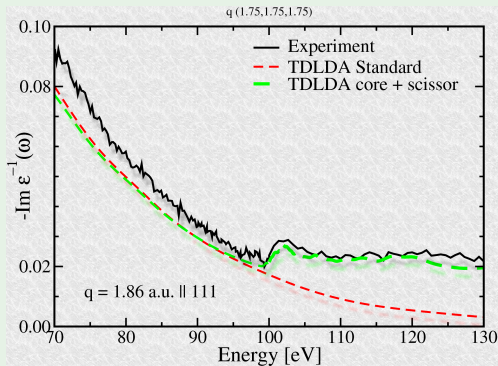
- transition-based analysis
- graphical tools
- real, imaginary part analysis

Limits

- 100-1000 atoms
- 0-100 eV

Semi-core states

L-edge of Silicon



Luppi *et al.* Phys. Rev. B **78**, 245124 (2008)

Outline

Information about the Beamline

Calculation(microscopic)-Measurement(macroscopic) Connection

Underlying theory and approximations

Usefulness of Theory

Codes

The Codes

- www.abinit.org *abinit.org*



- www.dp-code.org

- www.bethe-salpeter.org



- www.yambo-code.org

