

MBPT vs (TD)DFT

a fight or a wedding ?

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Outline

- 1 Introduction
- 2 BSE and TDDFT up to 2002
- 3 The Mapping Theory Kernel
 - Theory
 - Results
- 4 Conclusions and Perspectives

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A rough Summary

DFT - TDDFT

- ✓ fast (one-particle eqs)
- ✗ lack of functionals

MBPT (GW-BSE)

- ✓ it works!
(physical ingredients)
- ✗ Cumbersome

Fast, efficient and reliable

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A possible strategy

Combine the two approaches

A possible strategy

Dyson eq. for G

$$d \left[G = G_0 + G_0 (\Sigma - V_{xc}) G \right]$$

$$0 = G_0 (\Sigma - V_{xc}) G$$

$$\int d(23) G_0(12) [\Sigma(23) - V_{xc}(2) \delta(23)] G(31) = 0$$

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given a Σ (non-local and dynamic),
we obtain a V_{xc} (local and static) which provides the
same density

Sham-Schlüter equation



L.J.Sham and M.Schlter, Phys. Rev. Lett. **51**, 1888 (1983)

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we obtain $V_{xc}^{EXX} = G_0 G_0 v G_0 (\chi^0)^{-1}$

Exact-Exchange Approximation

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A possible strategy

Generalized SSE

$$p \left[G = G_0 + G_0 (\Sigma - V_{xc}) G \right]$$

 M.Gatti *et al*, Phys. Rev. Lett. **99**, 057401 (2007)

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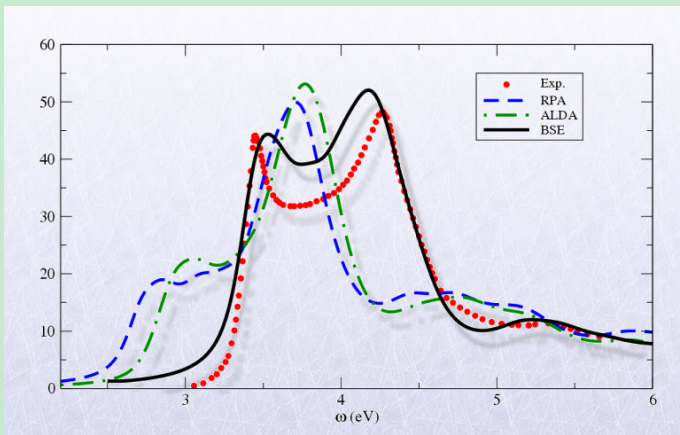
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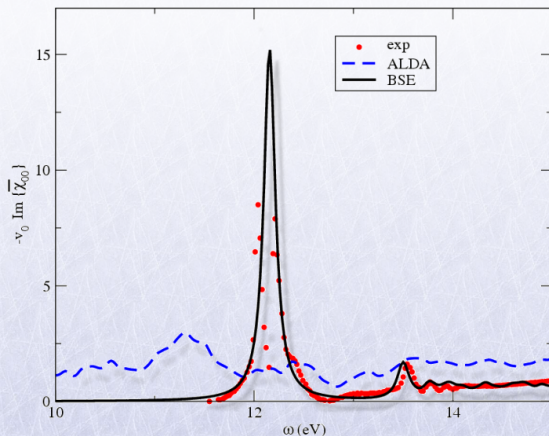
Optical Absorption Spectra of Solids

Semiconductors: Silicon



Optical Absorption Spectra of Solids

Insulators: Argon








Optical Absorption Spectra of Solids

- ALDA bad for any solids!! though quick
- BSE good but cumbersome

Optical Absorption Spectra of Solids






The problem of Abs in solids. Towards a better understanding

-  Reining *et al.* Phys.Rev.Lett. **88**, 66404 (2002)
Long-range kernel
-  de Boeij *et al.* J.Chem.Phys. **115**, 1995 (2002)
Polarization density functional. Long-range.
-  Kim and Görling Phys.Rev.Lett. **89**, 96402 (2002)
Exact-exchange
-  Sottile *et al.* Phys.Rev.B **68**, 205112 (2003)
Long-range and contact exciton.
-  Botti *et al.* Phys. Rev. B **72**, 125203 (2005)
Dynamic long-range component

Parameters to fit to experiments.

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Beyond ALDA approximation

Abs in solids. Insights from MBPT

Parameter-free **Ab initio** kernels

 Sottile *et al.* Phys.Rev.Lett. **91**, 56402 (2003)

Full many-body kernel. Mapping Theory.

 Marini *et al.* Phys.Rev.Lett. **91**, 256402 (2003)

Full many-body kernel. Perturbation Theory.

$$f_{xc} = \chi_0^{-1} GGWGG \chi_0^{-1}$$



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The Mapping Theory

The idea

BSE works \Rightarrow $\left\{ \begin{array}{l} \text{we get the ingredients of the BSE} \\ \text{and we put them in TDDFT} \end{array} \right.$



The Mapping Theory

BSE: Excitonic Hamiltonian

4-point

$$H_{(vc)(v'c')}^{\text{BSE}} = \left[(E_c - E_v) \delta_{vv'} \delta_{cc'} + v_{vc}^{v'c'} - W_{vc}^{v'c'} \right]$$



The Mapping Theory

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$$H^{\text{BSE}} = \left[(E_c - E_v) + \ll v \gg - \ll W \gg \right]$$



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$$H^{\text{BSE}} = \left[\left(\epsilon_c + \Delta_c^{\text{GW}} - \epsilon_v - \Delta_v^{\text{GW}} \right) + \ll v \gg - \ll W \gg \right]$$



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TDDFT: Polarizability equation

2-point

$$\chi = \chi_0 + \chi_0 (v + f_{xc}) \chi$$



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$$H^{\text{BSE}} = \left[\left(\epsilon_c + \Delta_c^{\text{GW}} - \epsilon_v - \Delta_v^{\text{GW}} \right) + \ll v \gg - \ll W \gg \right]$$

TDDFT: written in transition space

4-point

$$H^{\text{TDDFT}} = \left[\left(\epsilon_c - \epsilon_v \right) + \ll v \gg + \ll f_{xc} \gg \right]$$



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$$H^{\text{TDDFT}} = \left[(\epsilon_c - \epsilon_v) + \ll v \gg + \ll f_{\text{xc}} \gg \right]$$

The exchange-correlation kernel f_{xc} has to take into account both GW corrections and excitonic effects !!



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**Same starting point for both BSE and TDDFT:
the GW band-structure.**



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TDDFT: written in transition space

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$$H^{\text{TDDFT}} = \left[(E_c - E_v) + \ll v \gg + \ll f_{xc} \gg \right]$$

We concentrate, then, only on the excitonic effects.



The Mapping Theory

BSE: Excitonic Hamiltonian

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$$H^{\text{BSE}} = \left[(E_c - E_v) + \ll v \gg - \ll W \gg \right]$$

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$$H^{\text{TDDFT}} = \left[(E_c - E_v) + \ll v \gg - \ll W \gg \right]$$

We substitute the 'unknown' $\ll f_{xc} \gg$ with $\ll W \gg$.



The Mapping Theory

The idea

We want to use $\ll W \gg$, but in a 2-point equation.

$$\chi(12, \omega) = \chi_0(12, \omega) + \chi_0(13, \omega) (v(34) + f_{xc}(34, \omega)) \chi(42, \omega)$$



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$$\chi = \chi_0 + \chi_0 (\mathbf{v} + \mathbf{f}_{xc}) \chi$$

$$\chi = (\mathbf{1} - \chi_0 \mathbf{v} - \chi_0 \mathbf{f}_{xc})^{-1} \chi_0$$

Let's define an invertible matrix $X(12, \omega) = \sum_{vc} \phi_v(1) \phi_c(1) g_{vc}(2, \omega)$

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$$T(12, \omega) = X(13, \omega) f_{xc}(34, \omega) X(42, \omega) =$$

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$$T(12, \omega) = \sum_{\substack{vc \\ v'c'}} g_{vc}(1, \omega) \ll f_{xc} \gg g_{v'c'}(2, \omega)$$

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TDDFT 2-point equation containing $\ll W \gg$

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What about the application ??



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$$X(12, \omega) = \sum_{vc} \phi_v(1) \phi_c(1) g_{vc}(2, \omega)$$

What about the application ??



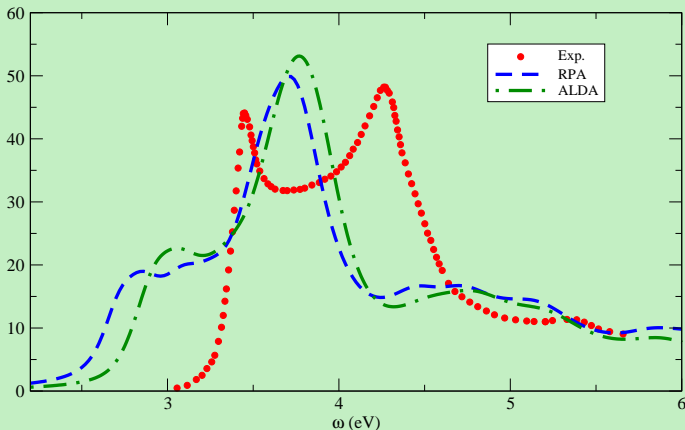
Outline

- 1 Introduction
- 2 BSE and TDDFT up to 2002
- 3 The Mapping Theory Kernel
 - Theory
 - Results
- 4 Conclusions and Perspectives



The Mapping Theory: Results

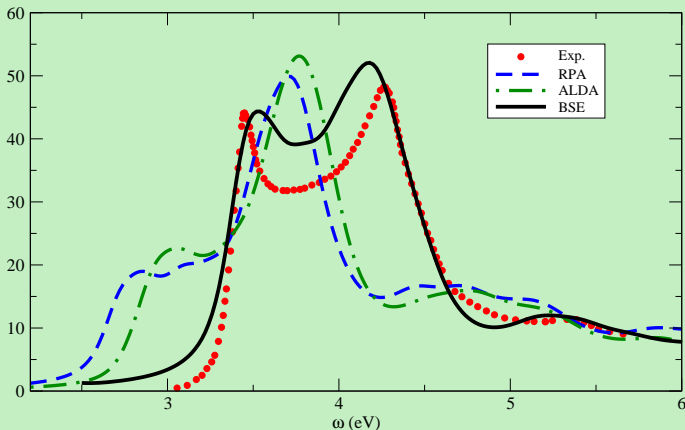
Absorption of Silicon





The Mapping Theory: Results

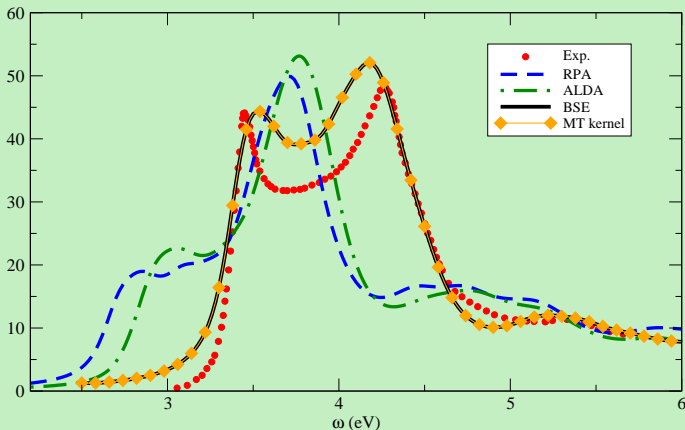
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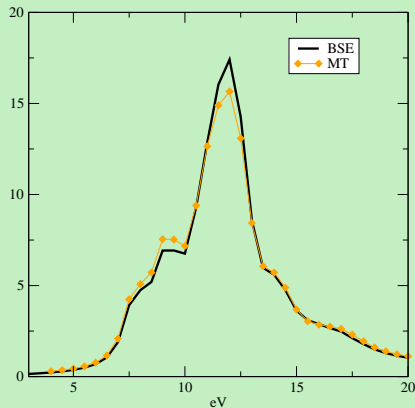
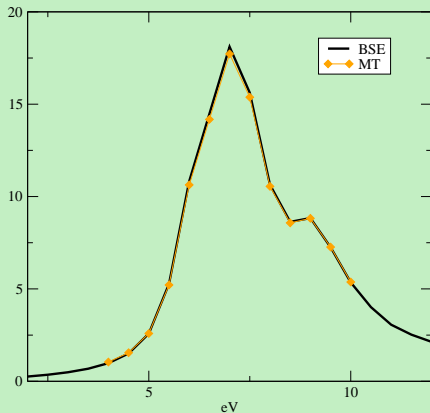


F.Sottile *et al.* Phys.Rev.Lett **91**, 56402 (2003)



The Mapping Theory: Results

Absorption of Silicon Carbide and Diamond

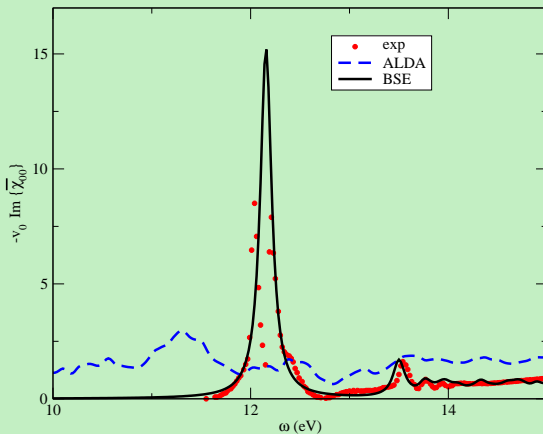


last week preliminary results :-)



The Mapping Theory: Results

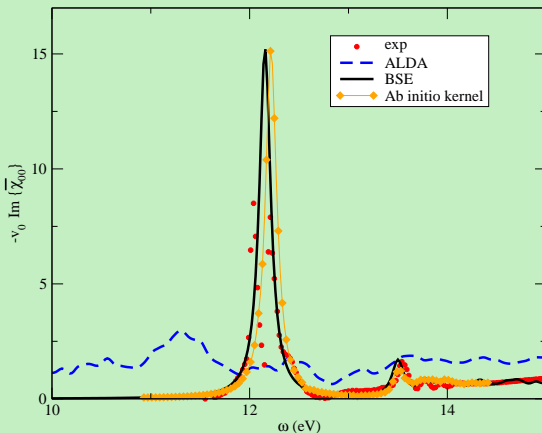
Absorption of Argon





The Mapping Theory: Results

Absorption of Argon







F.Sottile, M.Marsili *et al.*, PRB(R) **76**, 161103 (2007)



The Mapping Theory: Results

Tested also on absorption of SiO_2 , DNA bases, Ge-nanowires, RAS of diamond surface, and EELS of LiF.

-  Marini *et al.* Phys.Rev.Lett. **91**, 256402 (2003).
-  Bruno *et al.* Phys.Rev.B **72** 153310, (2005).
-  Palumbo *et al.* Phys.Rev.Lett. **94** 087404 (2005).
-  Varsano *et al.* J.Phys.Chem.B **110** 7129 (2006).

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Conclusions

TDDFT is the method of choice

- ✓ Absorption spectra of simple molecules
- ✓ Electron energy loss spectra
- ✓ Inelastic X-ray scattering spectroscopy
- ✓ Absorption of Solids (BSE-like scaling)

Conclusions

DFT-MBPT

- ⇒ Mapping Theory
- ⇒ OEP (EXX, etc.)

Functionals [ρ]

- ⇒ Meta-GGA
- ⇒ Orbital dependency

Extensions of TDDFT

- ⇒ TD-CDFT
- ⇒ Deformation Theory

Today challenges

- ⇒ Open shells systems
- ⇒ Charge transfer excitations
- ⇒ Efficient calculations of Solids

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