

Photoelectron Spectroscopy

Claudia Rödl and Matteo Gatti

Laboratoire des Solides Irradiés
Ecole Polytechnique, Palaiseau, France

SOLEIL Theory Day, Gif-sur-Yvette
May 6th, 2014



Methods: Starting from First Principles

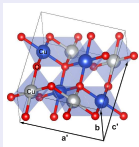
First-principles calculations

Density-functional theory (DFT)

Many-body perturbation theory (MBPT)

Ground state

- structural properties
- total energies
- charge densities



One-particle excitations

- band gaps
- band structures
- densities of states

Two-particle excitations:

- optical absorption
- excitonic states
- loss spectra

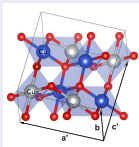
Methods: Starting from First Principles

First-principles calculations

Density-functional theory (DFT)

Ground state

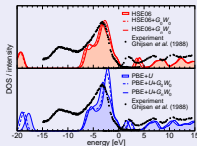
- structural properties
- total energies
- charge densities



Many-body perturbation theory (MBPT)

One-particle excitations

- band gaps
- band structures
- densities of states



Two-particle excitations:

- optical absorption
- excitonic states
- loss spectra

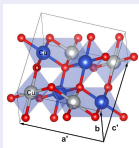
Methods: Starting from First Principles

First-principles calculations

Density-functional theory (DFT)

Ground state

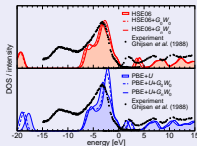
- structural properties
- total energies
- charge densities



Many-body perturbation theory (MBPT)

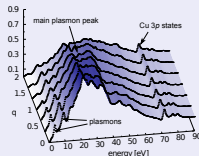
One-particle excitations

- band gaps
- band structures
- densities of states



Two-particle excitations:

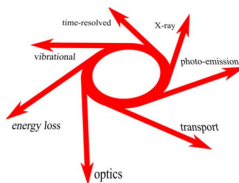
- optical absorption
- excitonic states
- loss spectra



ETSF Photoemission Beamline

Systems:

- insulators
- semiconductors
- metals
- correlated oxides



Codes:



Dimensions:

- bulk
 - surfaces
 - molecules
 - nanosystems
- ➔ up to ~ 100 atoms

ETSF Photoemission Beamline

Simulating photoemission

Spectral function

Beyond spectral function

Quasiparticle (QP)
approximation:

- QP band structures
- QP band gaps
- DOS

➡ *GW method*

Beyond QPs:

- lifetimes
- satellites
(e.g. plasmon
satellites)

➡ *GW method*

➡ *cumulant expansion*

Towards reality:

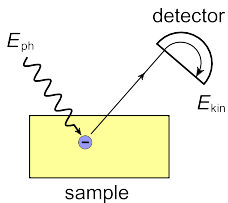
- matrix-element effects
(photoionization
cross-sections)
- extrinsic and
interference effects
- background

Photoemission and the Many-Body Problem

➔ probe the electronic structure

$$E_0^{(N)} + E_{\text{ph}} = E_n^{(N-1)} + E_{\text{kin}} + \Phi$$

$$E_{\text{ph}} = \underbrace{E_n^{(N-1)} - E_0^{(N)}}_{E_{\text{B}}} + E_{\text{kin}} + \Phi$$

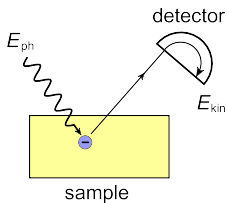
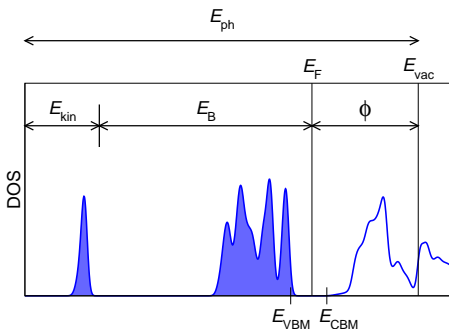


Photoemission and the Many-Body Problem

➔ probe the electronic structure

$$E_0^{(N)} + E_{\text{ph}} = E_n^{(N-1)} + E_{\text{kin}} + \Phi$$

$$E_{\text{ph}} = \underbrace{E_n^{(N-1)} - E_0^{(N)}}_{E_{\text{B}}} + E_{\text{kin}} + \Phi$$

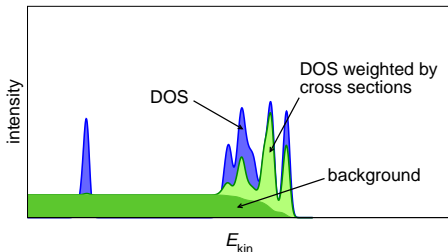
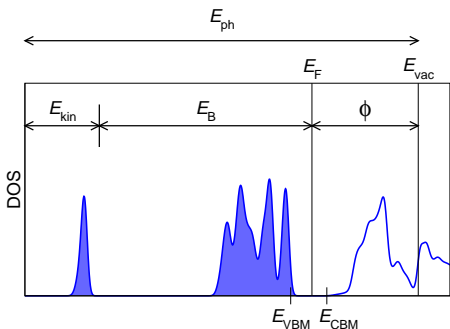
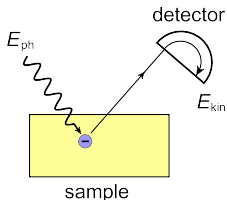


Photoemission and the Many-Body Problem

➔ probe the electronic structure

$$E_0^{(N)} + E_{\text{ph}} = E_n^{(N-1)} + E_{\text{kin}} + \Phi$$

$$E_{\text{ph}} = \underbrace{E_n^{(N-1)} - E_0^{(N)}}_{E_{\text{B}}} + E_{\text{kin}} + \Phi$$

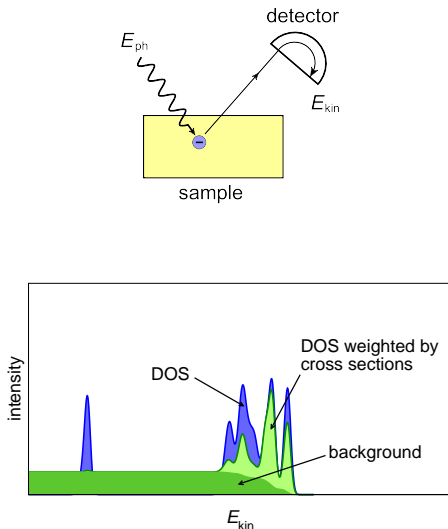
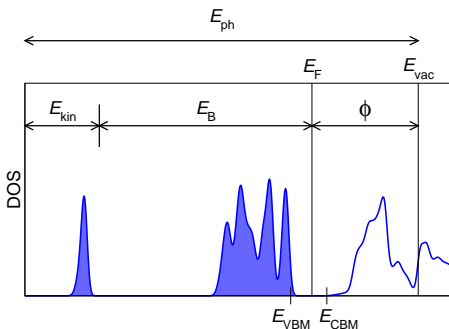


Photoemission and the Many-Body Problem

➔ probe the electronic structure

$$E_0^{(N)} + E_{\text{ph}} = E_n^{(N-1)} + E_{\text{kin}} + \Phi$$

$$E_{\text{ph}} = \underbrace{E_n^{(N-1)} - E_0^{(N)}}_{E_{\text{B}}} + E_{\text{kin}} + \Phi$$



➔ requires **full solution** of the **interacting many-body problem**: $E_0^{(N)}$, $E_n^{(N-1)}$

Key Quantity: Green's Function

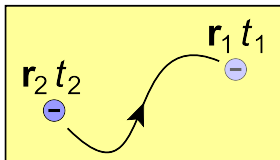
Green's function: $G(\mathbf{r}_1 t_1, \mathbf{r}_2 t_2) = \frac{1}{i\hbar} \langle N | \mathcal{T} \psi(\mathbf{r}_1 t_1) \psi^\dagger(\mathbf{r}_2 t_2) | N \rangle$

Key Quantity: Green's Function

$$\text{Green's function: } G(\mathbf{r}_1 t_1, \mathbf{r}_2 t_2) = \frac{1}{i\hbar} \langle N | \mathcal{T} \psi(\mathbf{r}_1 t_1) \psi^\dagger(\mathbf{r}_2 t_2) | N \rangle$$

Electron propagator:

$$\langle N | \psi(\mathbf{r}_1 t_1) \psi^\dagger(\mathbf{r}_2 t_2) | N \rangle$$



electron addition

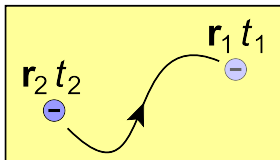
➔ inverse photoemission

Key Quantity: Green's Function

$$\text{Green's function: } G(\mathbf{r}_1 t_1, \mathbf{r}_2 t_2) = \frac{1}{i\hbar} \langle N | \mathcal{T} \psi(\mathbf{r}_1 t_1) \psi^\dagger(\mathbf{r}_2 t_2) | N \rangle$$

Electron propagator:

$$\langle N | \psi(\mathbf{r}_1 t_1) \psi^\dagger(\mathbf{r}_2 t_2) | N \rangle$$

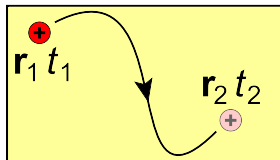


electron addition

➡ inverse photoemission

Hole propagator:

$$\langle N | \psi^\dagger(\mathbf{r}_2 t_2) \psi(\mathbf{r}_1 t_1) | N \rangle$$

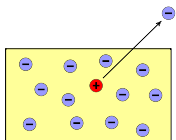


electron removal

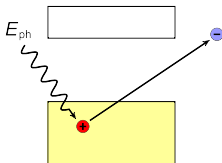
➡ photoemission

Spectral Function and Quasiparticles

real space

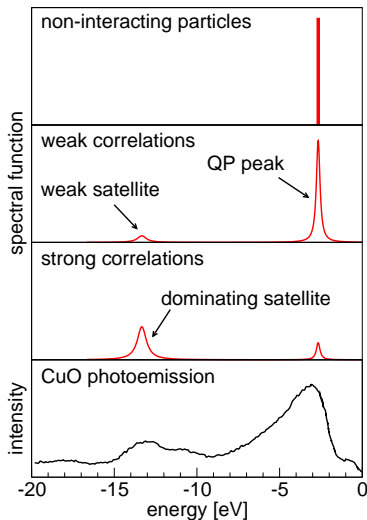


reciprocal space



Spectral function: $A(\omega) = \text{Tr} |\text{Im} G(\omega)|$

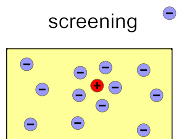
Density of states: $\text{DOS}(\omega) \sim A(\omega)$



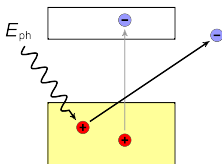
Experimental data:
Ghijssen *et al.*, PRB **38**, 11322 (1988)

Spectral Function and Quasiparticles

real space

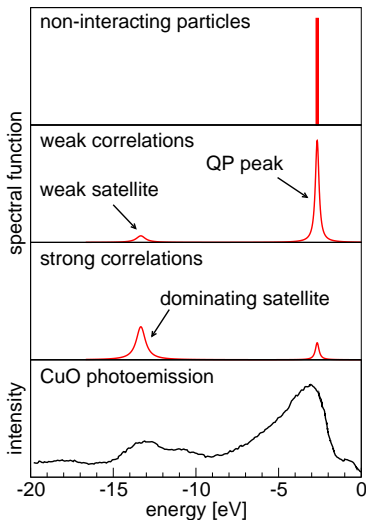


reciprocal space



Spectral function: $A(\omega) = \text{Tr} |\text{Im} G(\omega)|$

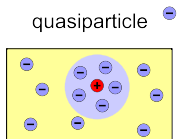
Density of states: $\text{DOS}(\omega) \sim A(\omega)$



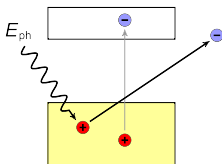
Experimental data:
Ghijssen *et al.*, PRB **38**, 11322 (1988)

Spectral Function and Quasiparticles

real space

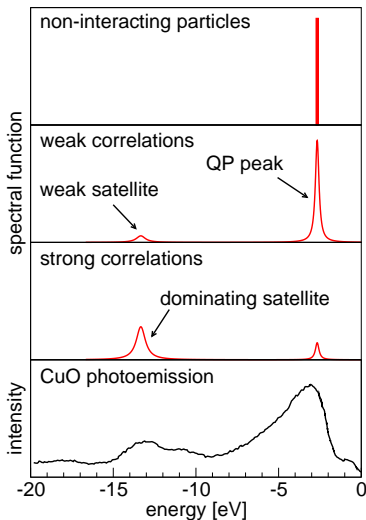


reciprocal space



Spectral function: $A(\omega) = \text{Tr} |\text{Im} G(\omega)|$

Density of states: $\text{DOS}(\omega) \sim A(\omega)$



Experimental data:
Ghijzen *et al.*, PRB **38**, 11322 (1988)

ETSF Photoemission Beamline

Simulating photoemission

Spectral function

Beyond spectral function

Quasiparticle (QP)
approximation:

- QP band structures
- QP band gaps
- DOS

➡ *GW method*

Beyond QPs:

- lifetimes
- satellites
(e.g. plasmon
satellites)

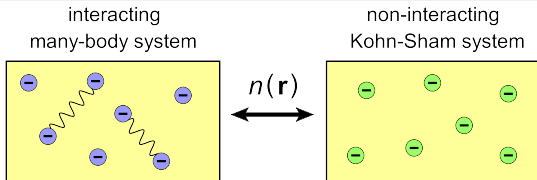
➡ *GW method*

➡ *cumulant expansion*

Towards reality:

- matrix-element effects
(photoionization
cross-sections)
- extrinsic and
interference effects
- background

Density-Functional Theory (DFT)



Kohn-Sham equations:

Kohn, Sham, Phys. Rev. **140**, A1133 (1965)

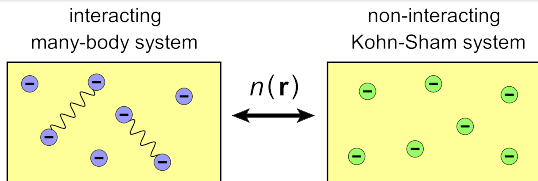
$$\left[-\frac{\hbar^2}{2m_0} \Delta + V_{\text{ext}}(\mathbf{r}) + V_{\text{H}}(\mathbf{r}, [n]) + V_{\text{XC}}(\mathbf{r}, [n]) \right] \varphi_{n\mathbf{k}}(\mathbf{r}) = \varepsilon_{n\mathbf{k}} \varphi_{n\mathbf{k}}(\mathbf{r})$$

density: $n(\mathbf{r}) = \sum_{n\mathbf{k}}^{\text{occ}} |\varphi_{n\mathbf{k}}(\mathbf{r})|^2$



self-consistent solution

Density-Functional Theory (DFT)



Kohn-Sham equations:

Kohn, Sham, Phys. Rev. **140**, A1133 (1965)

$$\left[-\frac{\hbar^2}{2m_0} \Delta + V_{\text{ext}}(\mathbf{r}) + V_{\text{H}}(\mathbf{r}, [n]) + V_{\text{XC}}(\mathbf{r}, [n]) \right] \varphi_{n\mathbf{k}}(\mathbf{r}) = \varepsilon_{n\mathbf{k}} \varphi_{n\mathbf{k}}(\mathbf{r})$$

density: $n(\mathbf{r}) = \sum_{n\mathbf{k}}^{\text{occ}} |\varphi_{n\mathbf{k}}(\mathbf{r})|^2$

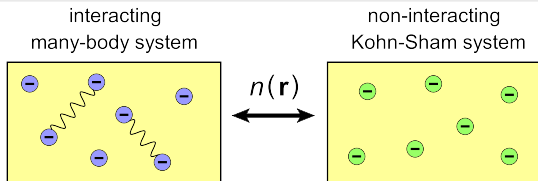


self-consistent solution

Exchange-correlation potential V_{XC} ?

- local potential $V_{\text{XC}}(\mathbf{r}, [n])$: LDA, PBE, ...
- non-local potential $V_{\text{XC}}(\mathbf{r}\mathbf{r}', [n])$:
 - hybrid functionals (PBE0, HSE06, ...)
 - on-site term U for localized orbitals (LDA+ U , ...)

Density-Functional Theory (DFT)



Kohn-Sham equations:

Kohn, Sham, Phys. Rev. **140**, A1133 (1965)

$$\left[-\frac{\hbar^2}{2m_0} \Delta + V_{\text{ext}}(\mathbf{r}) + V_{\text{H}}(\mathbf{r}, [n]) + V_{\text{XC}}(\mathbf{r}, [n]) \right] \varphi_{n\mathbf{k}}(\mathbf{r}) = \varepsilon_{n\mathbf{k}} \varphi_{n\mathbf{k}}(\mathbf{r})$$

density: $n(\mathbf{r}) = \sum_{n\mathbf{k}}^{\text{occ}} |\varphi_{n\mathbf{k}}(\mathbf{r})|^2$



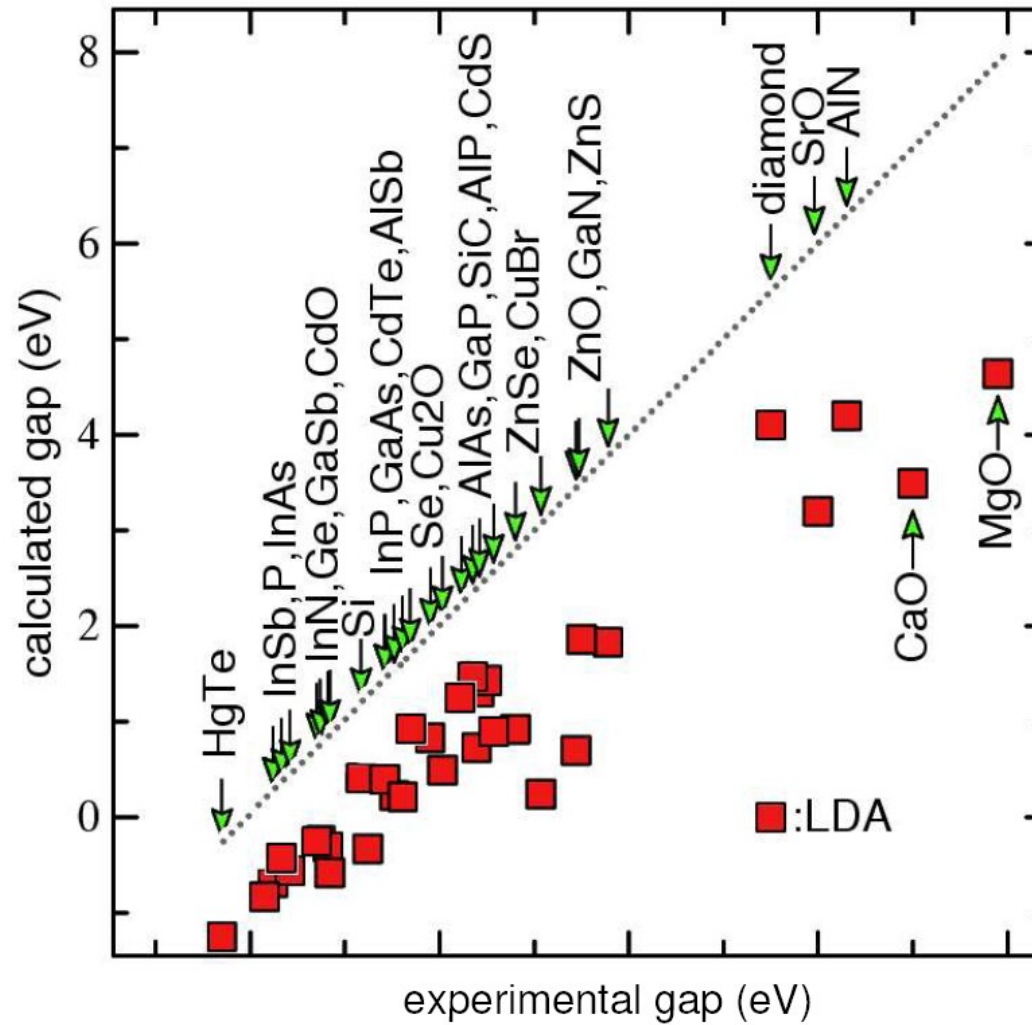
self-consistent solution

Problems:

- Kohn-Sham band structure has **no physical meaning** (strictly speaking)
- **band-gap problem**: strong underestimation of band gaps

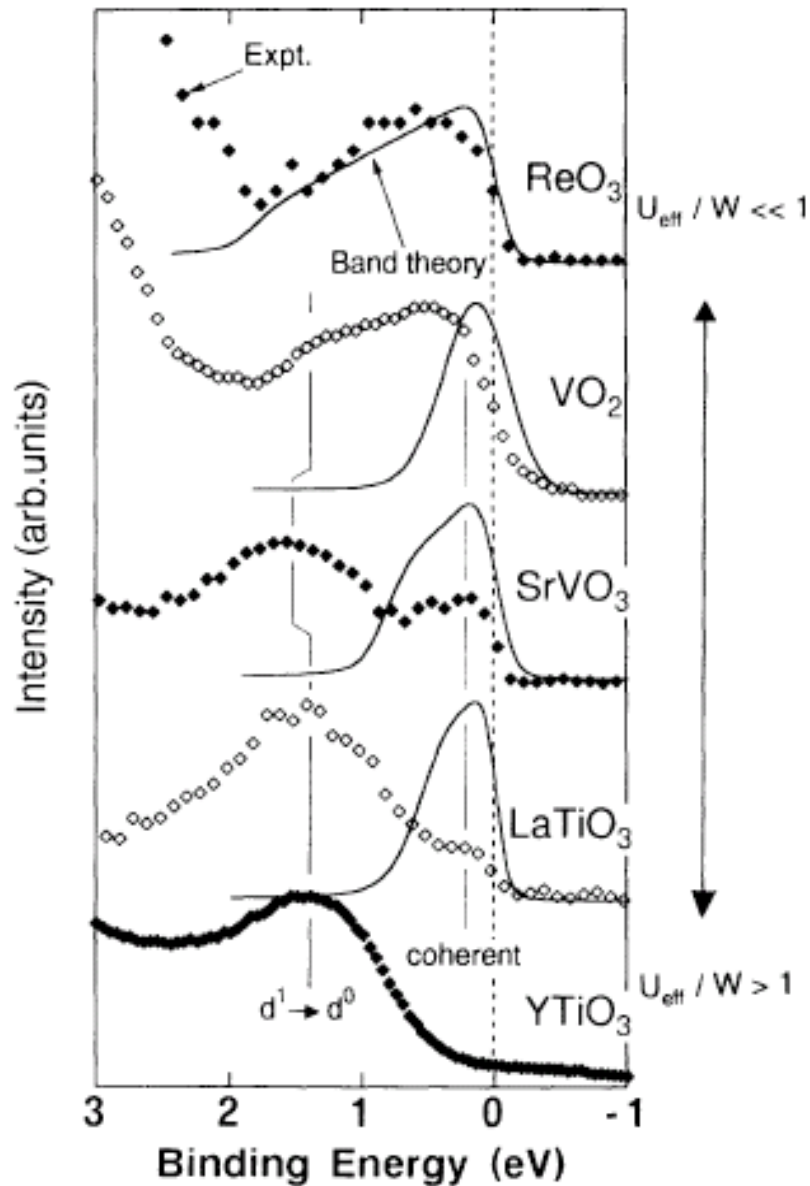
➡ electron addition and removal properties wrong

Why more than DFT?



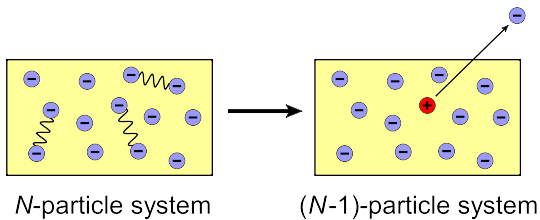
adapted from M. van Schilfgaarde et al., PRL 96 (2006)

Why more than DFT?

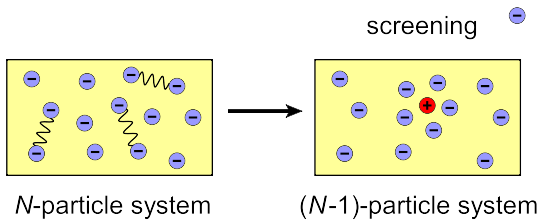


LDA = "band theory" ?

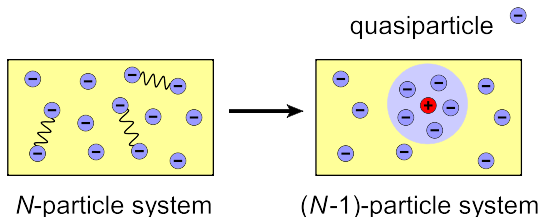
Quasiparticle Excitations



Quasiparticle Excitations



Quasiparticle Excitations



Quasiparticle equation:

$$\left[-\frac{\hbar^2}{2m_0} \Delta + V_{\text{ext}}(\mathbf{r}) + V_{\text{H}}(\mathbf{r}) \right] \varphi_{n\mathbf{k}}(\mathbf{r}) + \int d\mathbf{r}' \Sigma(\mathbf{r}\mathbf{r}', \omega = \varepsilon_{n\mathbf{k}}/\hbar) \varphi_{n\mathbf{k}}(\mathbf{r}') = \varepsilon_{n\mathbf{k}} \varphi_{n\mathbf{k}}(\mathbf{r})$$

Problems:

- exchange and correlation: non-local and frequency-dependent self-energy Σ
- self-consistent solution
- no explicit expression for the self-energy

Hedin's GW Approximation

Self-energy: $\Sigma \approx i\hbar G W$

Green's function $G(\mathbf{r}_1 t_1, \mathbf{r}_2 t_2)$

Dynamically screened
Coulomb interaction $W(\mathbf{r}_1 \mathbf{r}_2, \omega)$

Hedin, Phys. Rev. **139**, A796 (1965)

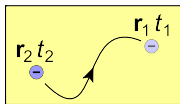
Hedin, Lundqvist, Solid State Phys. **23**, 1 (1969)

Hedin's GW Approximation

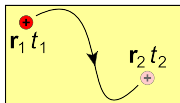
$$\text{Self-energy: } \Sigma \approx i\hbar GW$$

Green's function $G(\mathbf{r}_1 t_1, \mathbf{r}_2 t_2)$

Electron propag.: $\langle N | \psi(\mathbf{r}_1 t_1) \psi^\dagger(\mathbf{r}_2 t_2) | N \rangle$



Hole propag.: $\langle N | \psi^\dagger(\mathbf{r}_2 t_2) \psi(\mathbf{r}_1 t_1) | N \rangle$



Dynamically screened
Coulomb interaction $W(\mathbf{r}_1 \mathbf{r}_2, \omega)$

Hedin, Phys. Rev. **139**, A796 (1965)

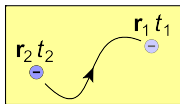
Hedin, Lundqvist, Solid State Phys. **23**, 1 (1969)

Hedin's GW Approximation

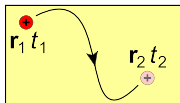
$$\text{Self-energy: } \Sigma \approx i\hbar GW$$

Green's function $G(\mathbf{r}_1 t_1, \mathbf{r}_2 t_2)$

Electron propag.: $\langle N | \psi(\mathbf{r}_1 t_1) \psi^\dagger(\mathbf{r}_2 t_2) | N \rangle$

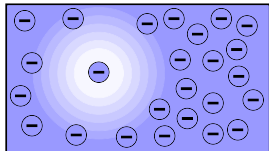


Hole propag.: $\langle N | \psi^\dagger(\mathbf{r}_2 t_2) \psi(\mathbf{r}_1 t_1) | N \rangle$



Dynamically screened
Coulomb interaction $W(\mathbf{r}_1 \mathbf{r}_2, \omega)$

$$W(\mathbf{r}_1 \mathbf{r}_2, \omega) = \int d\mathbf{r}_3 \varepsilon^{-1}(\mathbf{r}_1 \mathbf{r}_3, \omega) v(\mathbf{r}_3 - \mathbf{r}_2)$$



Hedin, Phys. Rev. **139**, A796 (1965)

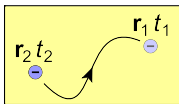
Hedin, Lundqvist, Solid State Phys. **23**, 1 (1969)

Hedin's GW Approximation

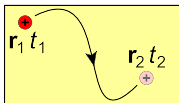
$$\text{Self-energy: } \Sigma \approx i\hbar GW$$

Green's function $G(\mathbf{r}_1 t_1, \mathbf{r}_2 t_2)$

Electron propag.: $\langle N | \psi(\mathbf{r}_1 t_1) \psi^\dagger(\mathbf{r}_2 t_2) | N \rangle$

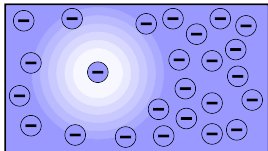


Hole propag.: $\langle N | \psi^\dagger(\mathbf{r}_2 t_2) \psi(\mathbf{r}_1 t_1) | N \rangle$



Dynamically screened
Coulomb interaction $W(\mathbf{r}_1 \mathbf{r}_2, \omega)$

$$W(\mathbf{r}_1 \mathbf{r}_2, \omega) = \int d\mathbf{r}_3 \varepsilon^{-1}(\mathbf{r}_1 \mathbf{r}_3, \omega) v(\mathbf{r}_3 - \mathbf{r}_2)$$

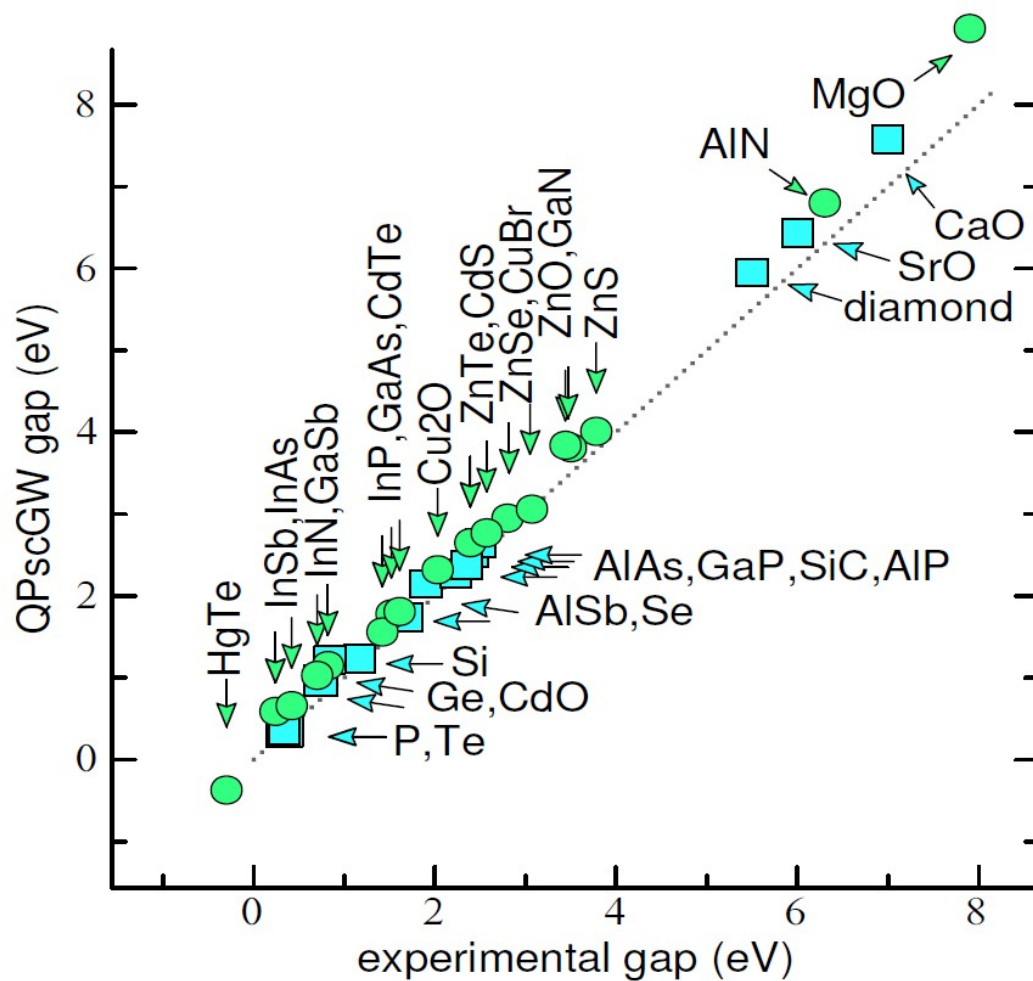


➔ main contribution:
dynamical screened exchange
(Hartree-Fock: $\varepsilon^{-1}(\mathbf{r}_1 \mathbf{r}_3, \omega) \equiv 1$)

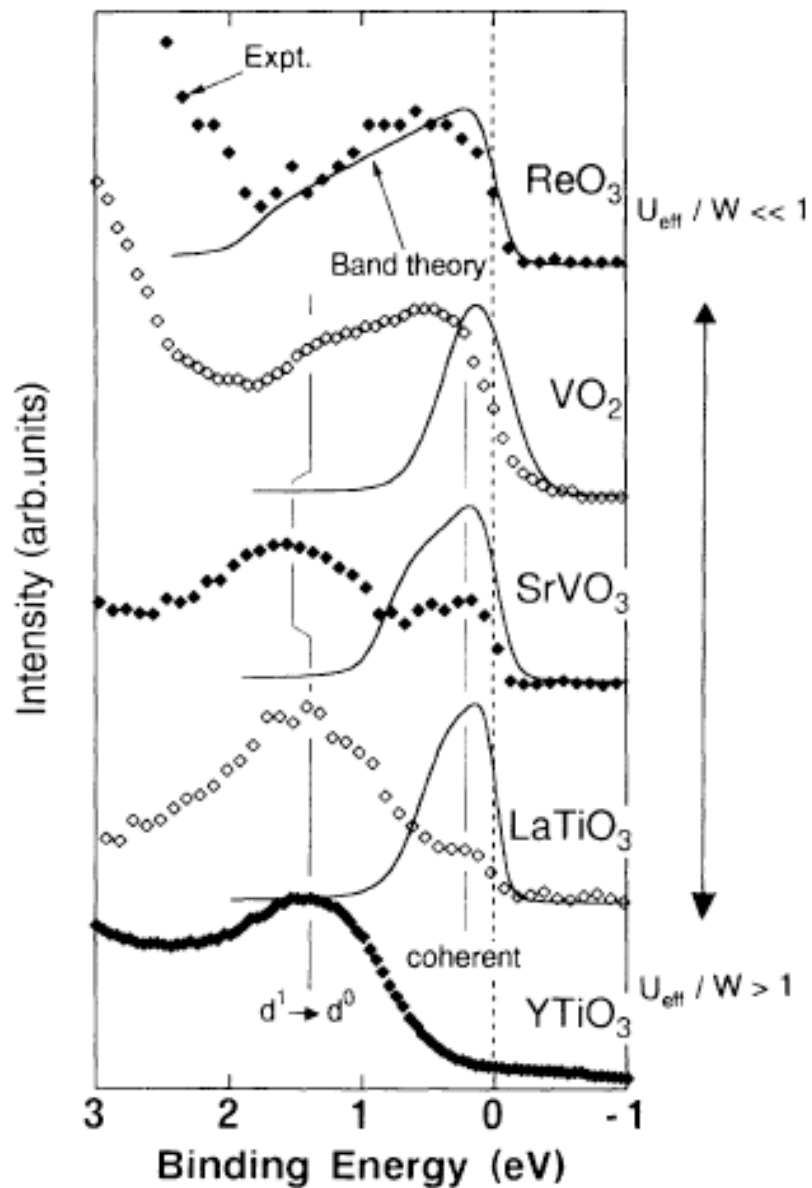
Hedin, Phys. Rev. **139**, A796 (1965)

Hedin, Lundqvist, Solid State Phys. **23**, 1 (1969)

Why more than DFT?

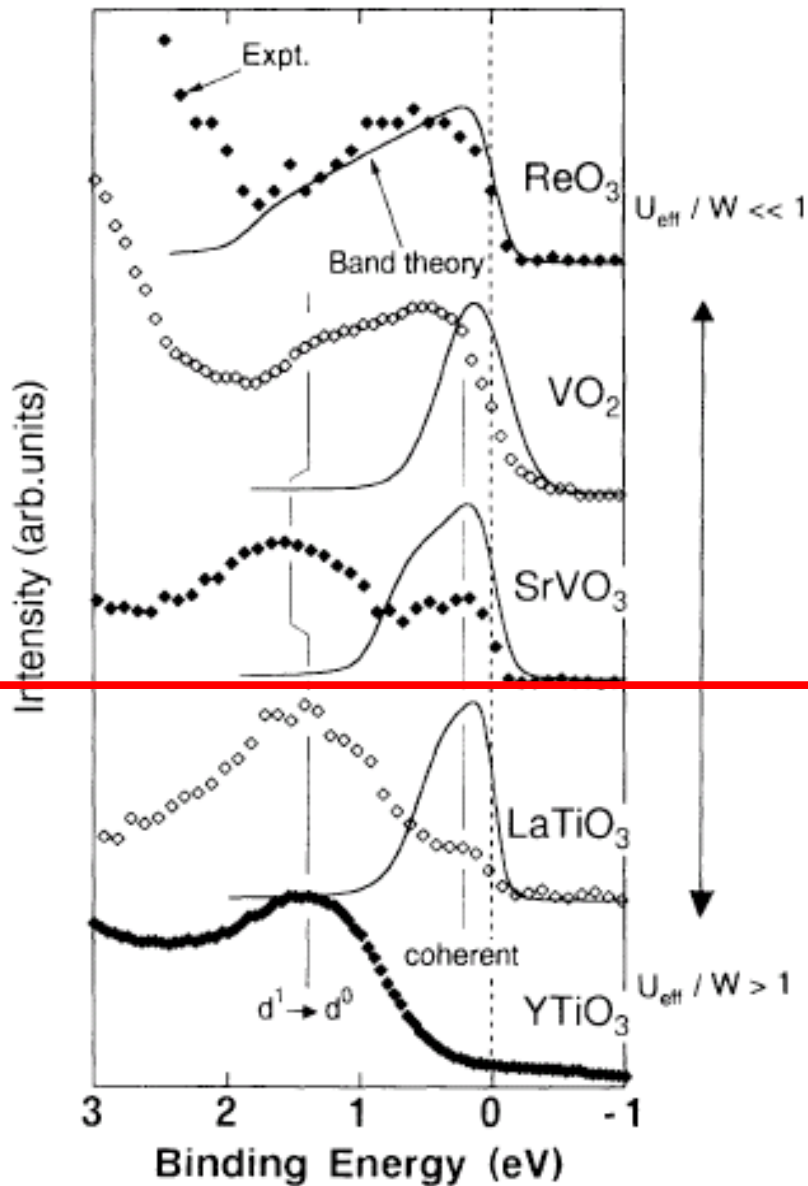


adapted from M. van Schilfgaarde et al., PRL 96 (2006)



LDA = "band theory" ?

Also missing physics!



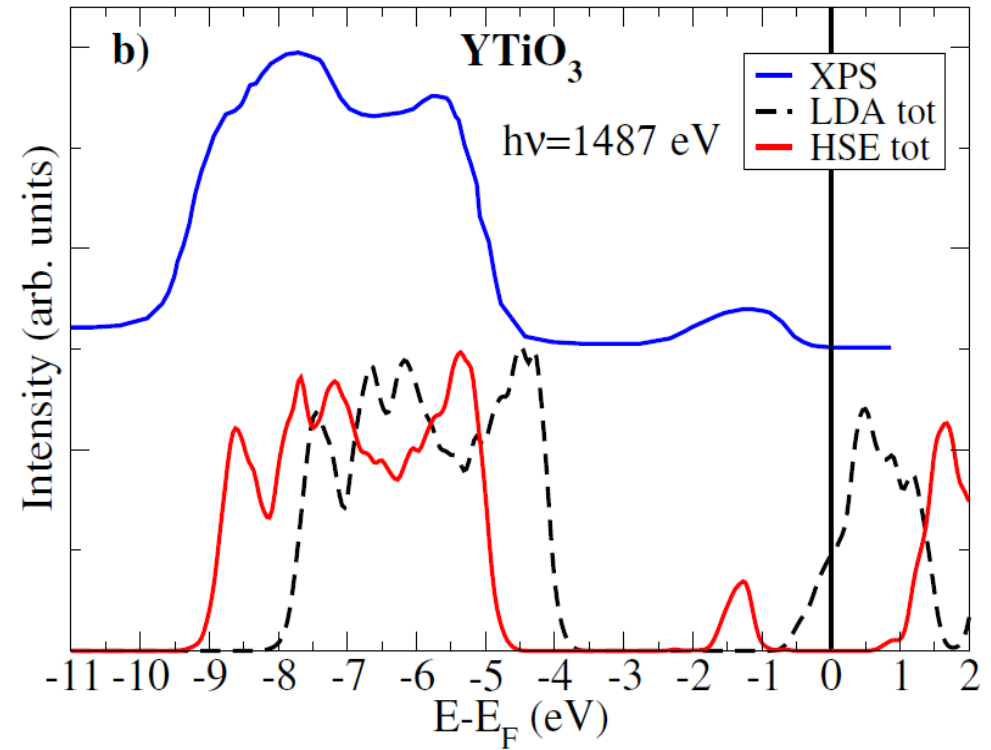
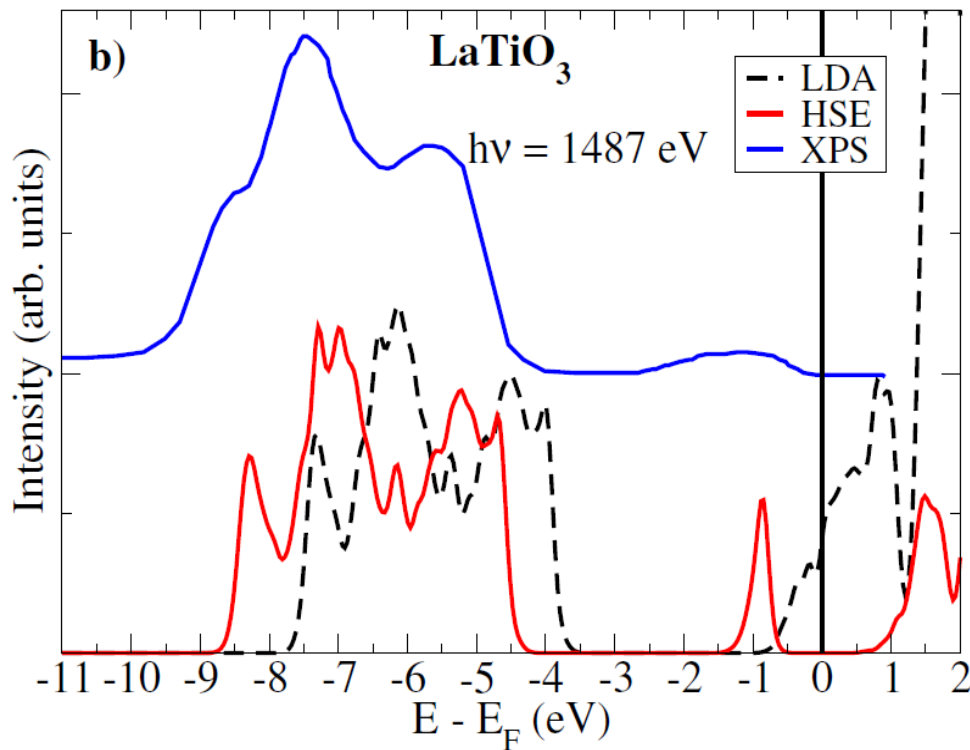
LDA = "band theory" ?

Also missing physics!

Exchange + Correlation

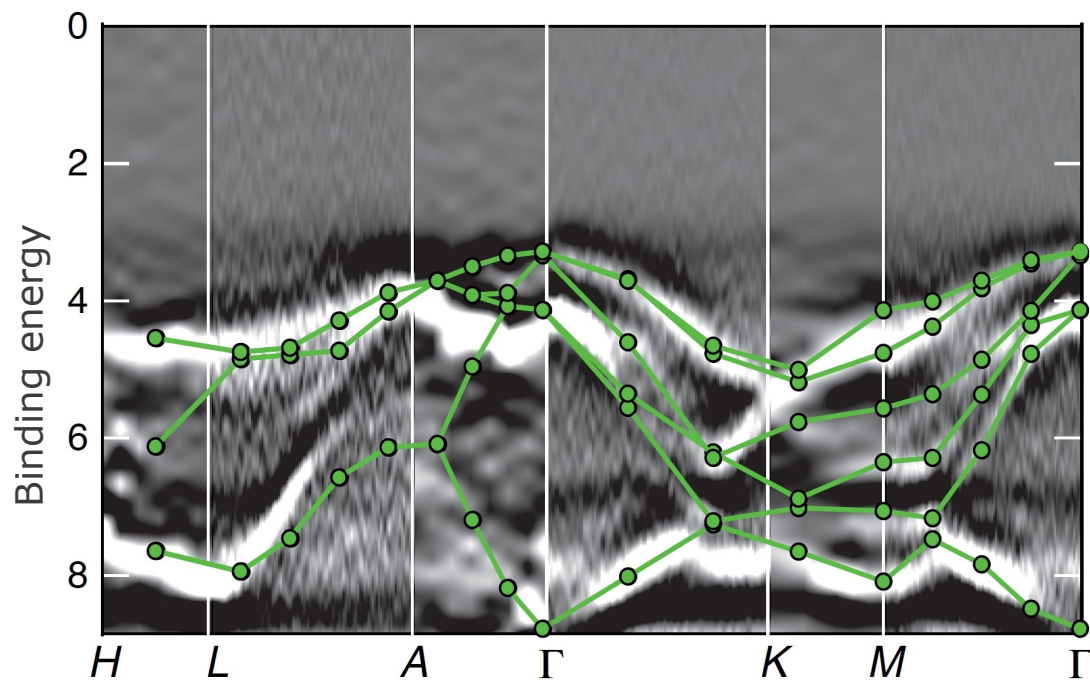
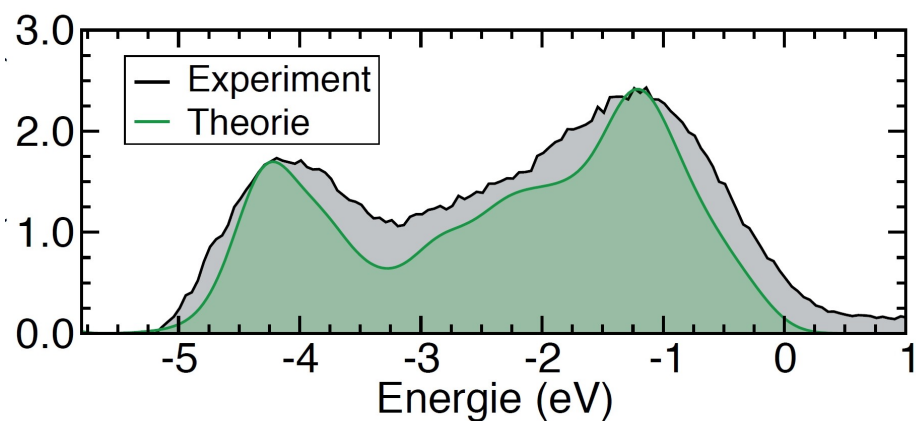
Mott insulators?

The role of nonlocal exchange



F. Iori, M. Gatti and A. Rubio, PRB 85 (2012)
Exp. from H. Roth PhD thesis – Köln 2008

ARPES vs. band structure: ZnO



Exp from M. Kobayashi, et al. Proceedings of the 29th International Conference on the Physics of Semiconductors (2008)
André Schleife PhD thesis, University of Jena (2010)

ETSF Photoemission Beamline

Simulating photoemission

Spectral function

Beyond spectral function

Quasiparticle (QP)
approximation:

- QP band structures
- QP band gaps
- DOS

➡ *GW method*

Beyond QPs:

- lifetimes
- satellites
(e.g. plasmon
satellites)

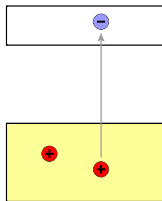
➡ *GW method*

➡ *cumulant expansion*

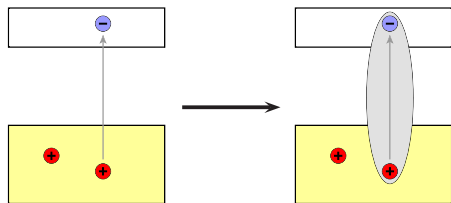
Towards reality:

- matrix-element effects
(photoionization
cross-sections)
- extrinsic and
interference effects
- background

Electron Correlation ... 3-Particle Problem

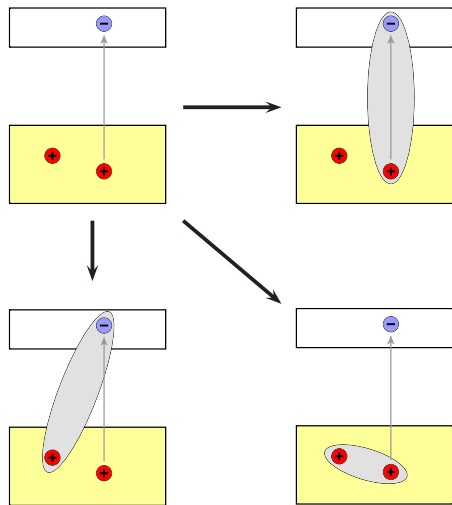


Electron Correlation ... 3-Particle Problem



- $\Sigma = i\hbar GW$
- plasmon satellite series
Langreth, PRB **1**, 471 (1970)
Aryasetiawan *et al.*,
PRL **77**, 2268 (1996)
Guzzo *et al.*,
PRL **107**, 166401 (2011)

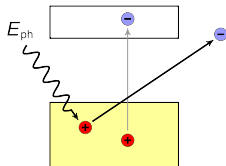
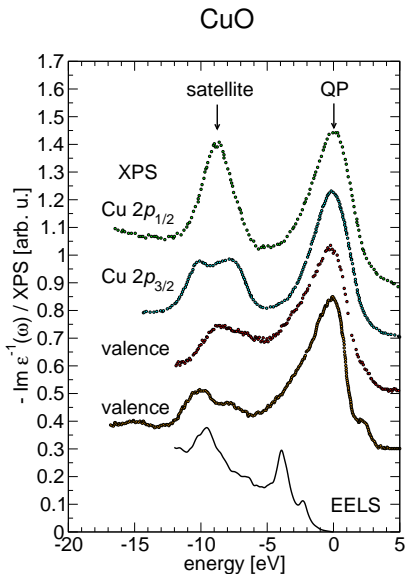
Electron Correlation ... 3-Particle Problem



- $\Sigma = i\hbar GW$
- plasmon satellite series
Langreth, PRB **1**, 471 (1970)
Aryasetiawan *et al.*,
PRL **77**, 2268 (1996)
Guzzo *et al.*,
PRL **107**, 166401 (2011)

- $\Sigma = i\hbar GT$
- hole-hole or electron-hole
 T matrix
- 6 eV satellite in Ni
Springer *et al.*, PRL **80**, 2389 (1998)

Spectral Function ... Plasmon Satellites



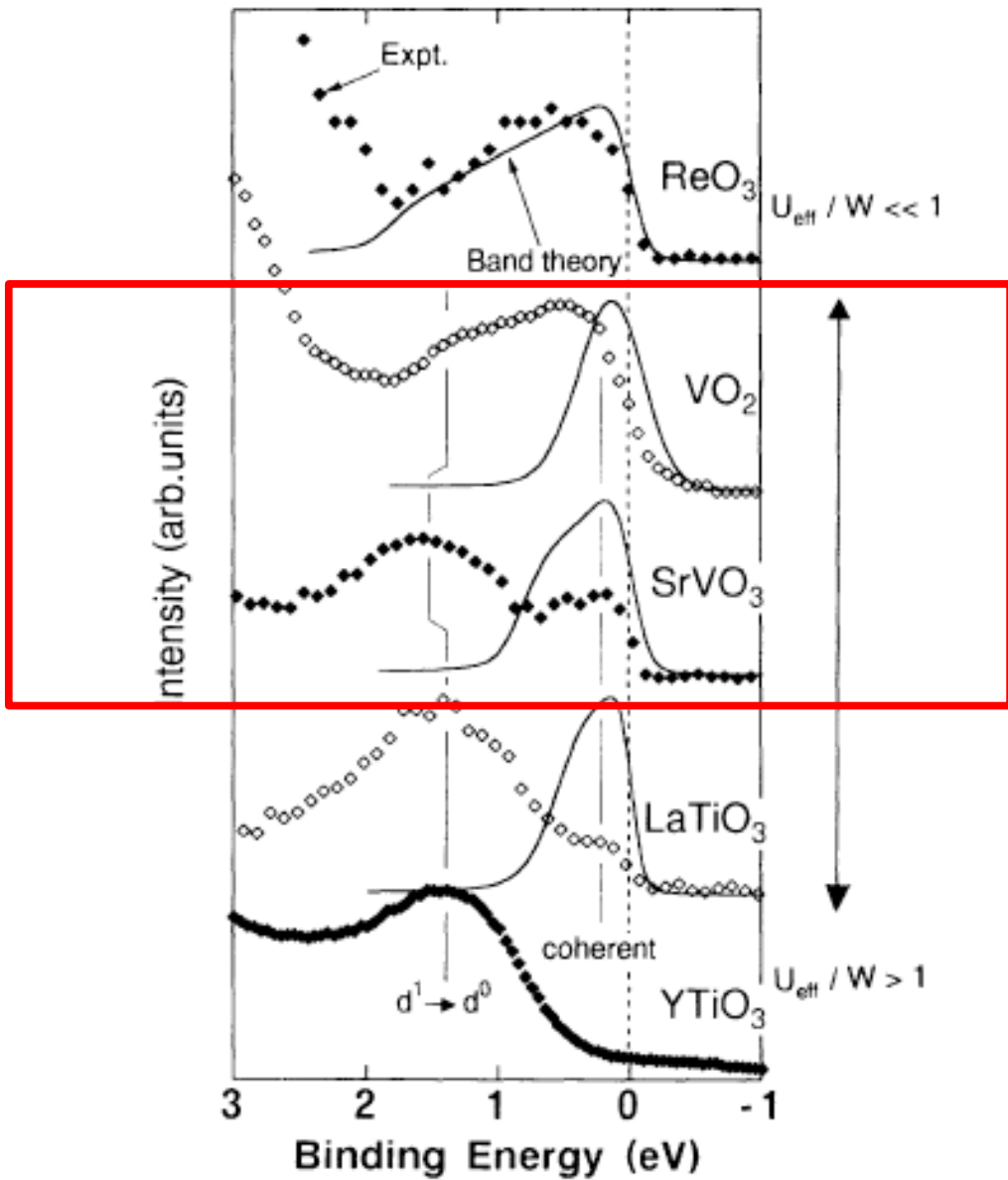
- **satellite**: response of the material to the photoemission hole remaining in the system
- ➔ **plasmon excitations**
- prominent loss peaks should appear in satellite structure

Experimental data:

Simons, PhD thesis (2008)

Ghijsen *et al.*, PRB **38**, 11322 (1988)

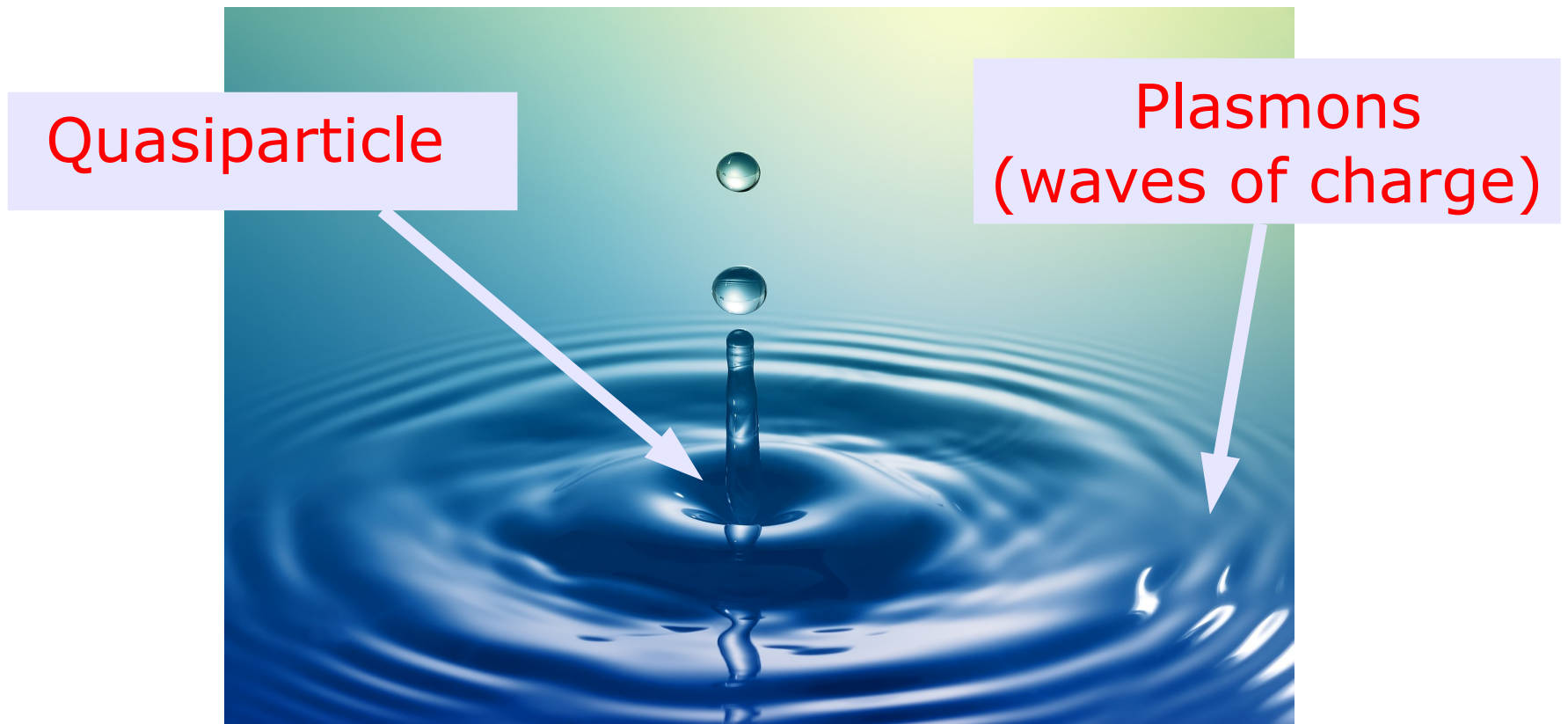
Shen *et al.*, PRB **42**, 8081 (1990)



Correlated metals:
Satellites?

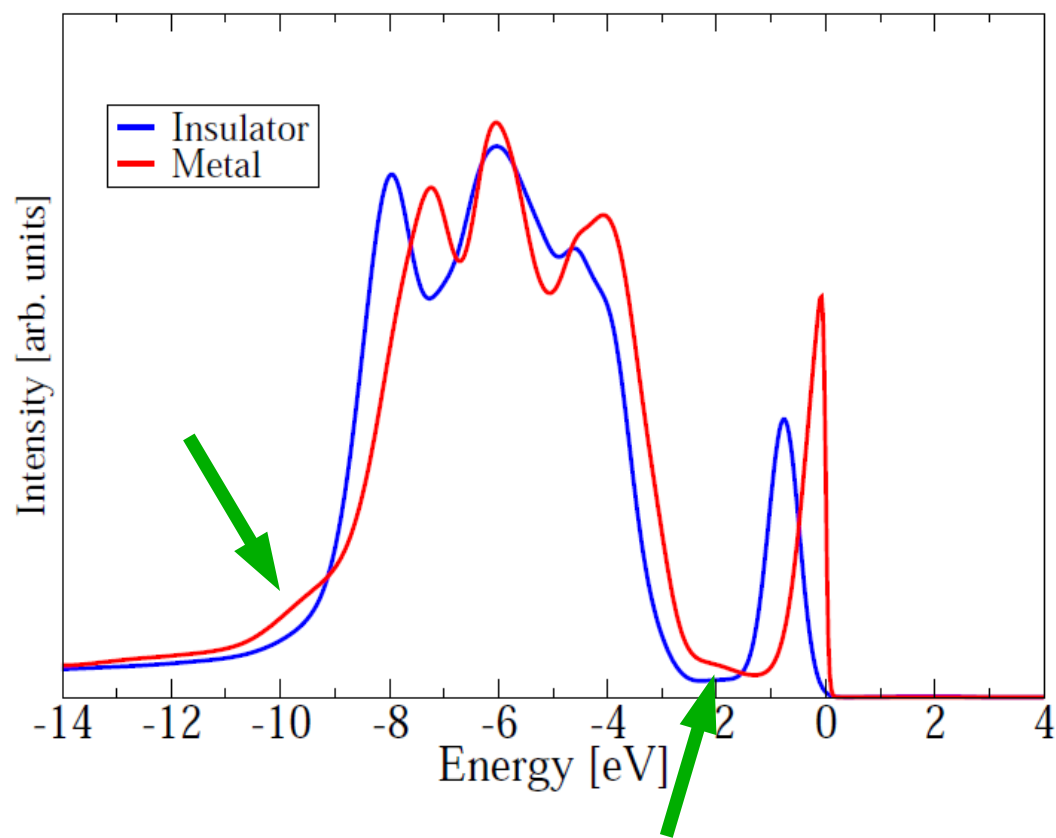
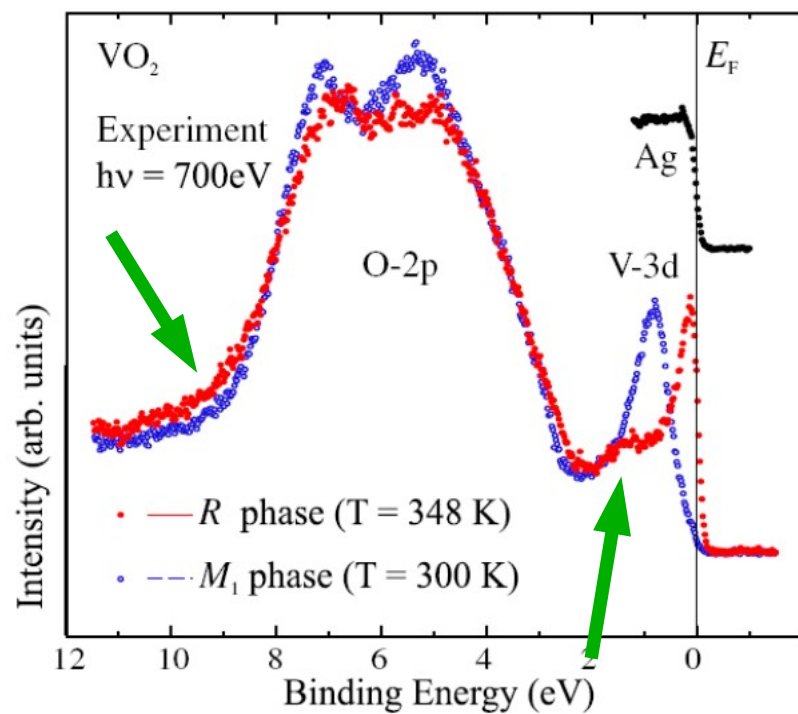
Correlation = coupling of excitations

Adding/removing electrons: reaction of all the others



Dynamical correlation: VO₂

Metal-Insulator transition in VO₂

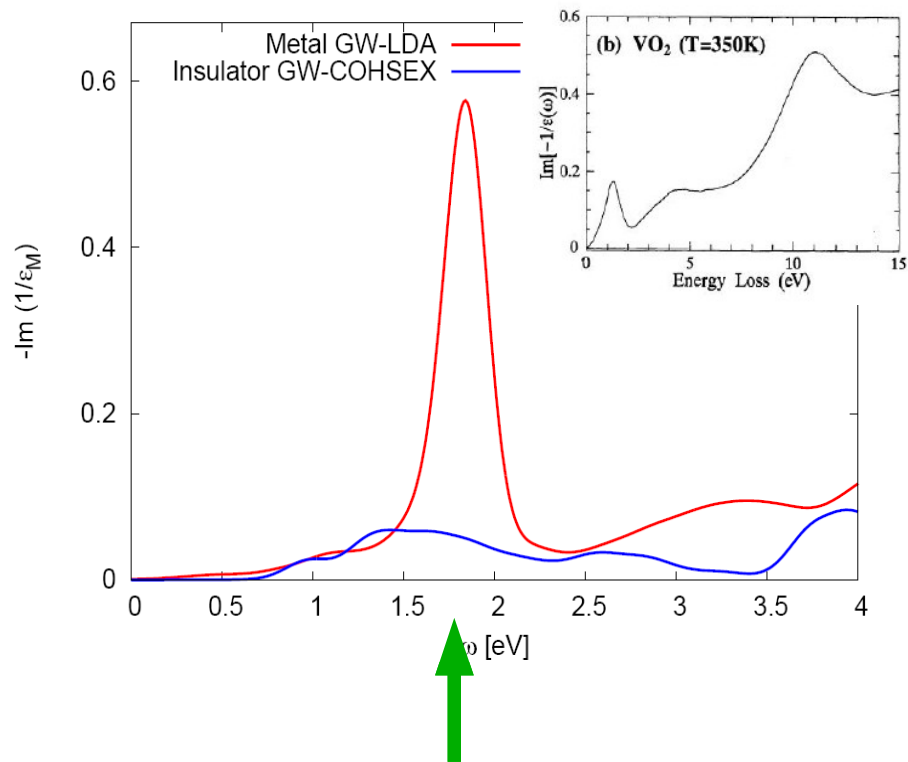


Spectral function

Dynamical correlation: VO₂

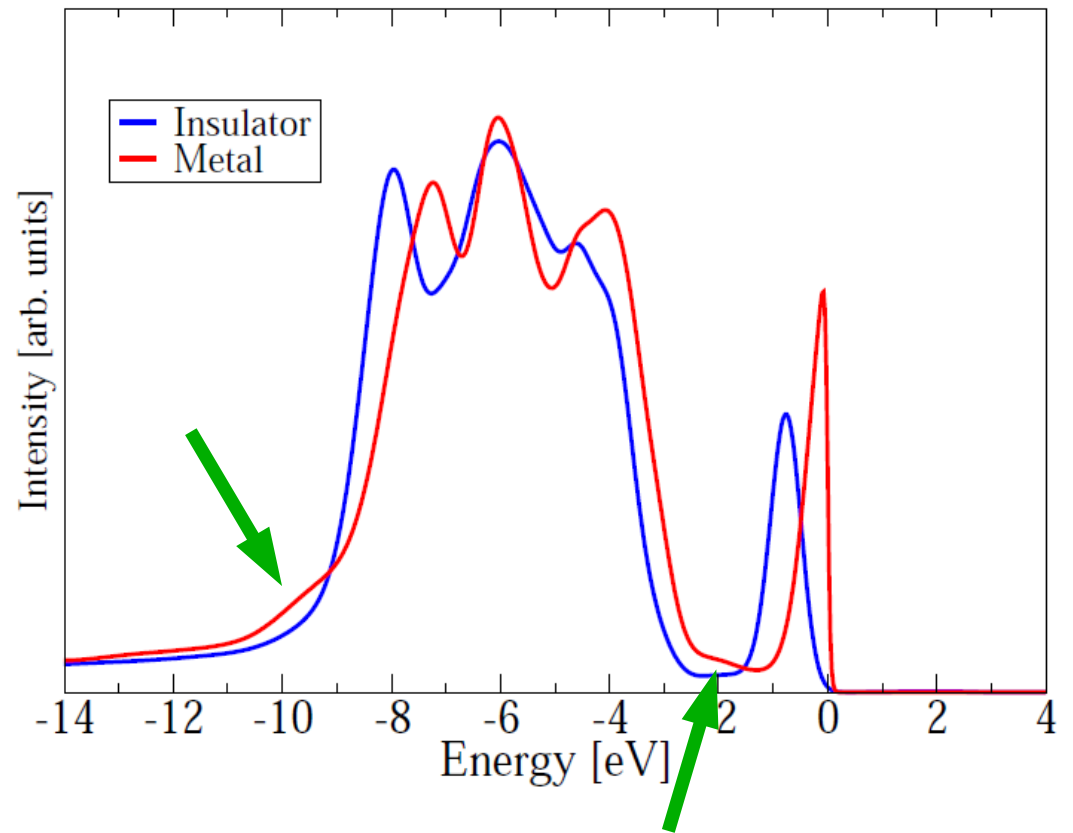
$-\text{Im } \epsilon^{-1}(q, \omega)$

IXS/EELS



$$W(r, r', \omega) = \epsilon^{-1}(r, r', \omega) v(|r - r'|)$$

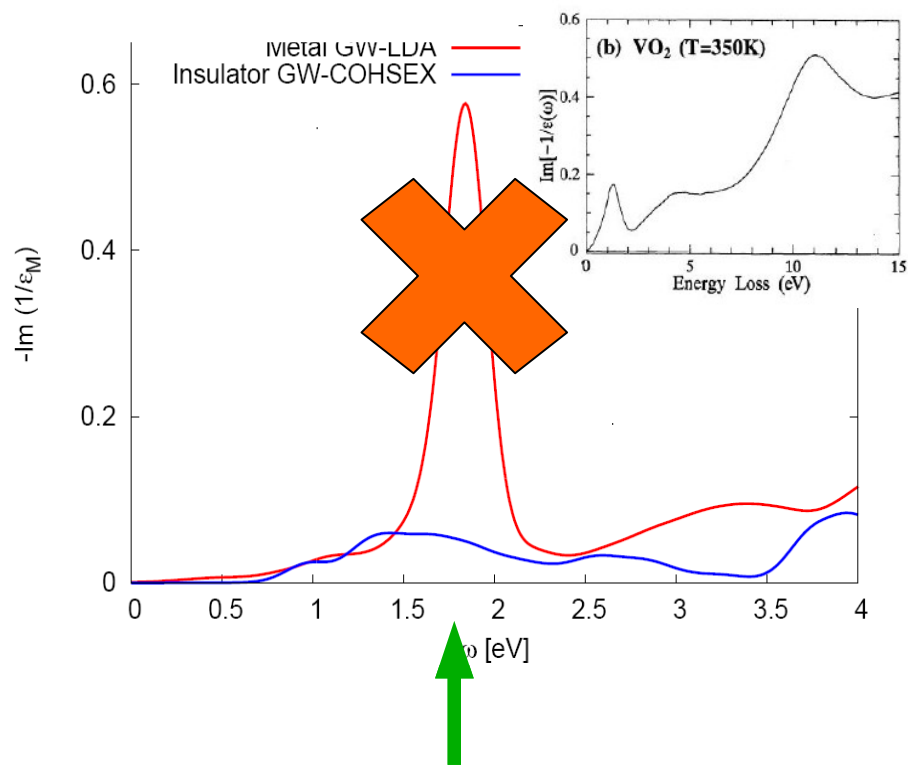
Metal-Insulator transition in VO₂



Spectral function

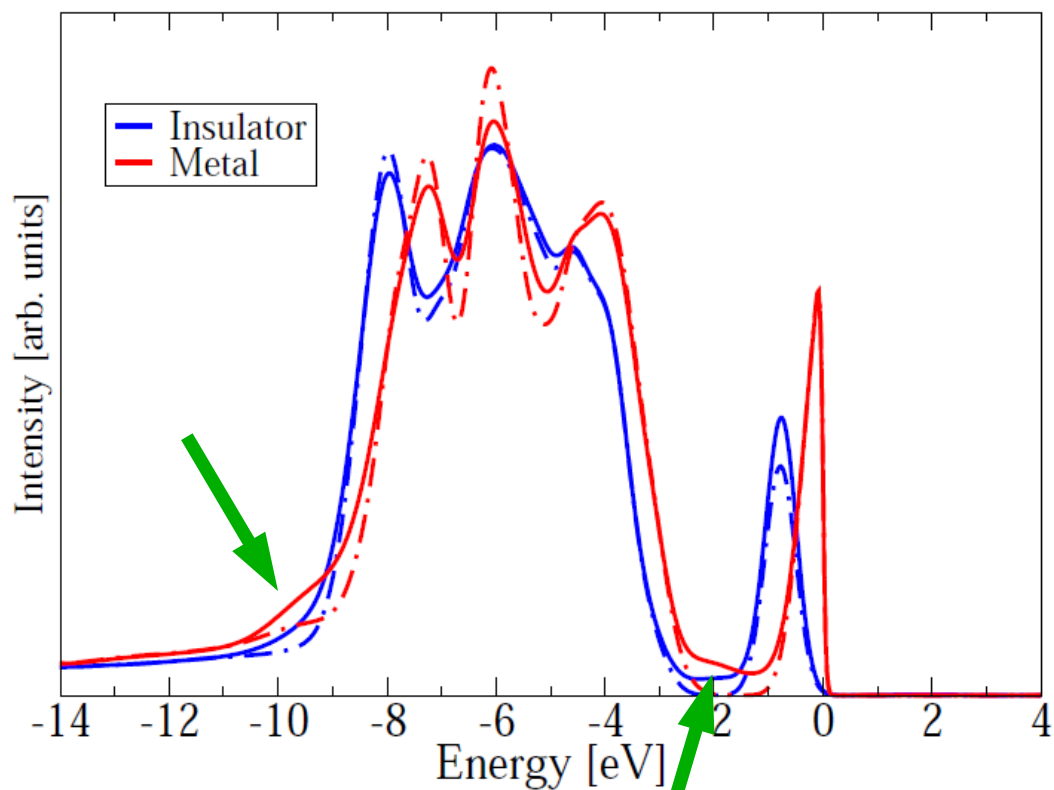
Dynamical correlation: VO₂

$-\text{Im } \epsilon^{-1}(q, \omega)$ **IXS/EELS**



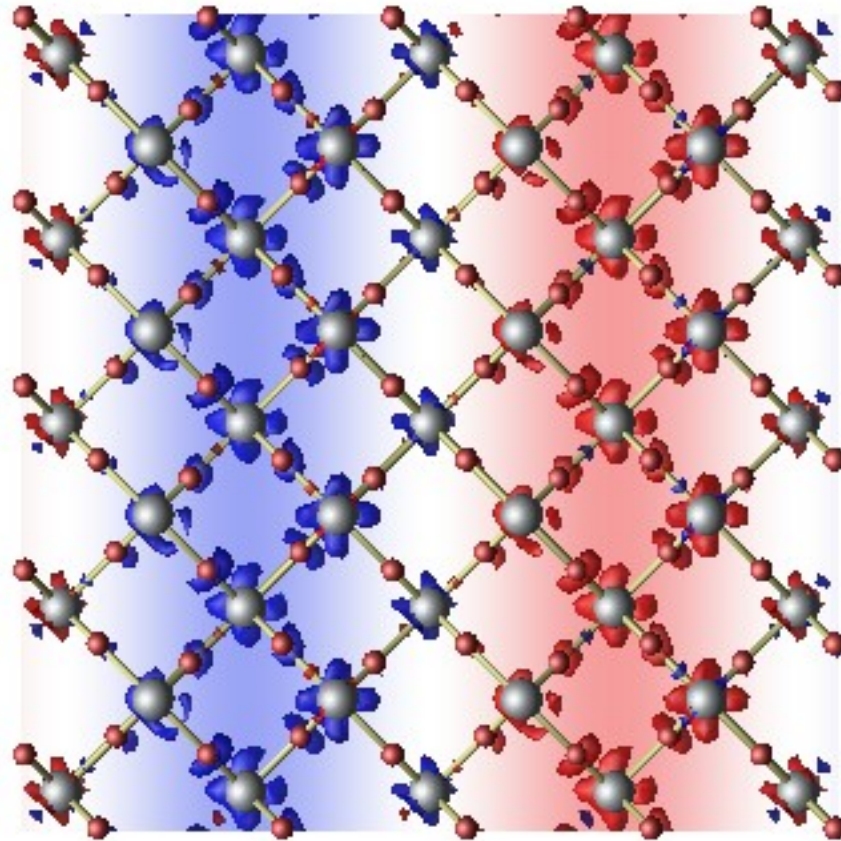
$$W(r, r', \omega) = \epsilon^{-1}(r, r', \omega) v(|r - r'|)$$

Metal-Insulator transition in VO₂



Spectral function

Dynamical correlation: VO_2



See Francesco's talk

ETSF Photoemission Beamline

Simulating photoemission

Spectral function

Beyond spectral function

Quasiparticle (QP)
approximation:

- QP band structures
- QP band gaps
- DOS

➡ *GW method*

Beyond QPs:

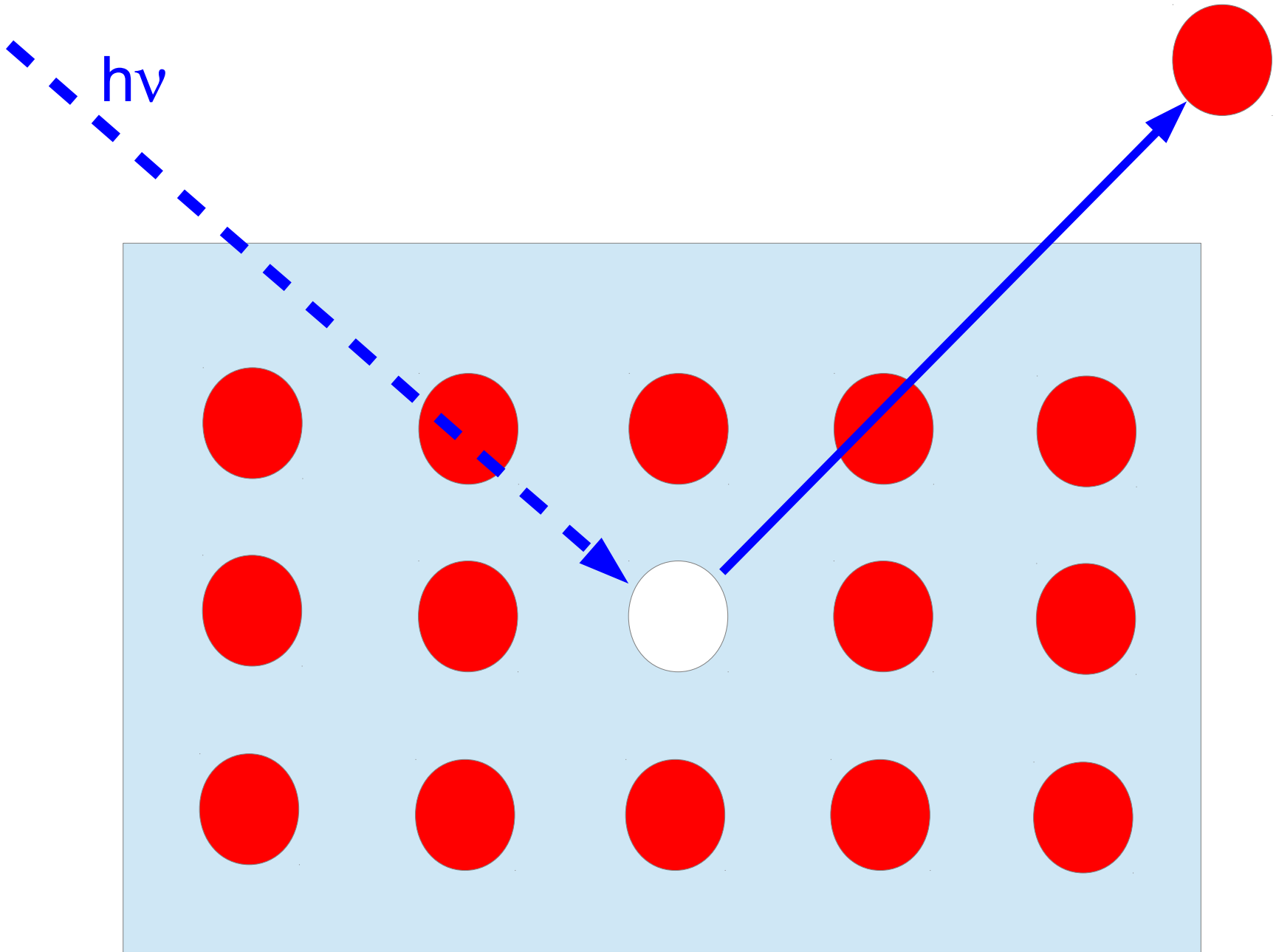
- lifetimes
- satellites
(e.g. plasmon
satellites)

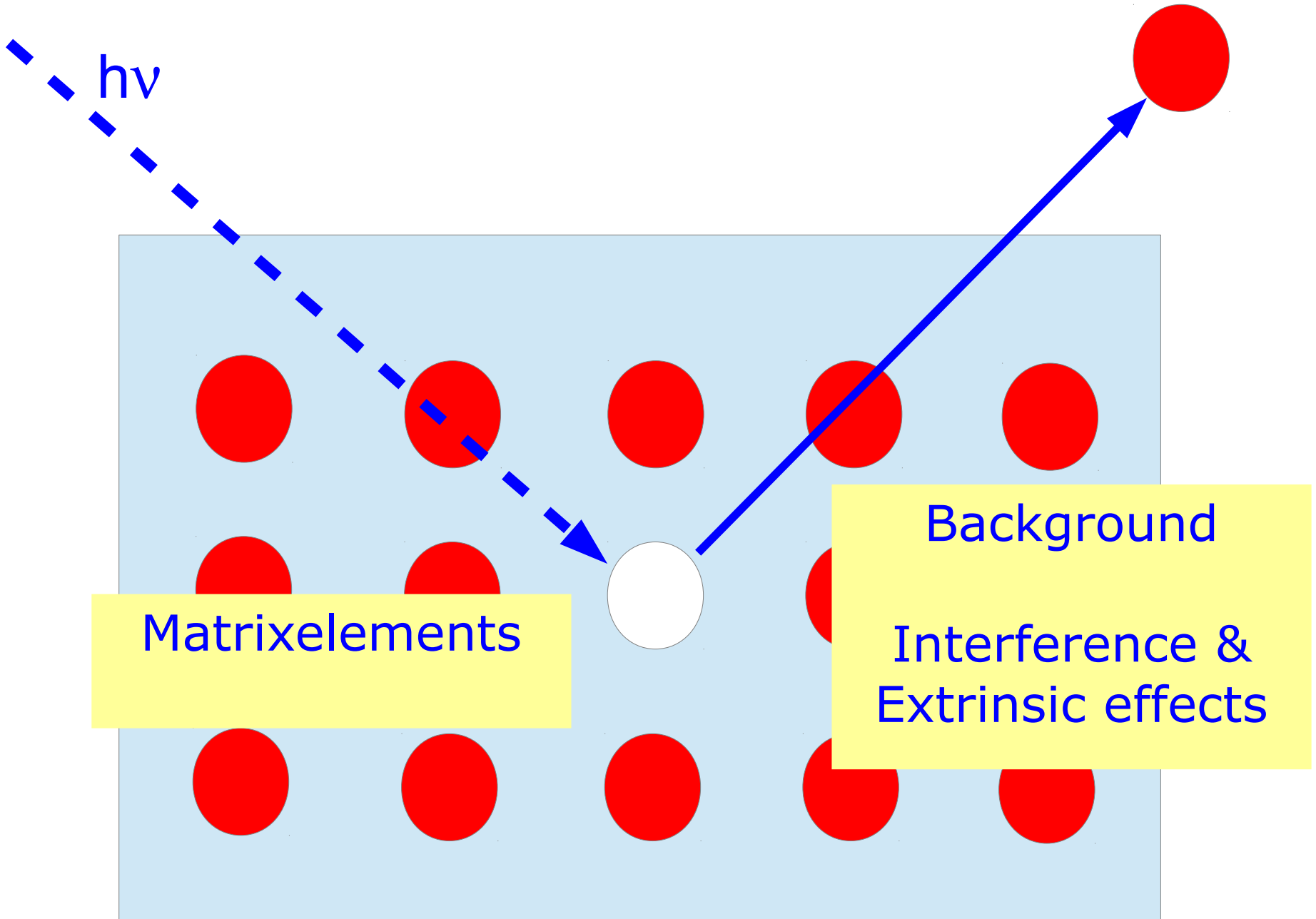
➡ *GW method*

➡ *cumulant expansion*

Towards reality:

- matrix-element effects
(photoionization
cross-sections)
- extrinsic and
interference effects
- background



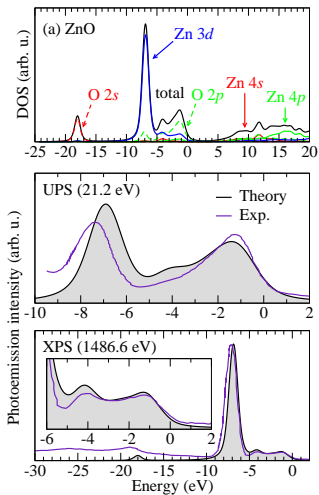


Matrixelements

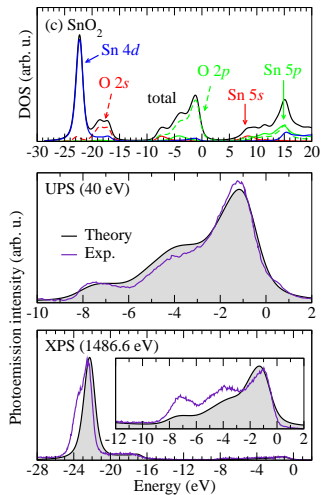
Background
Interference &
Extrinsic effects

TCOs: Including Photoionization Cross-Sections

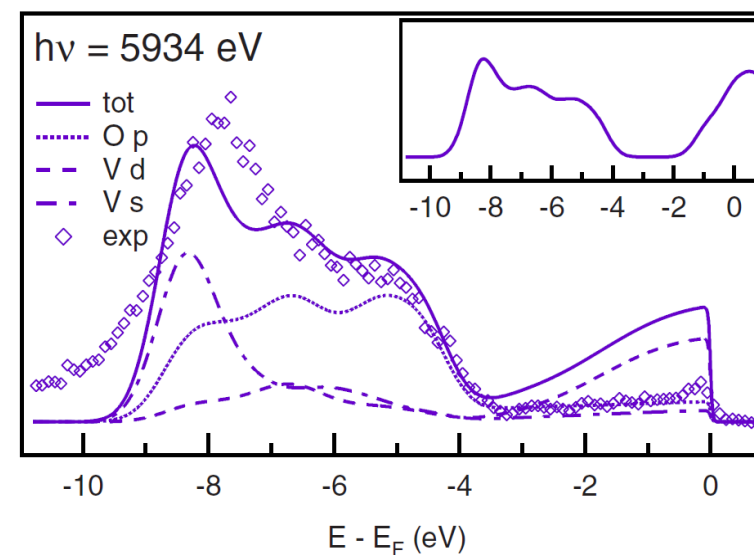
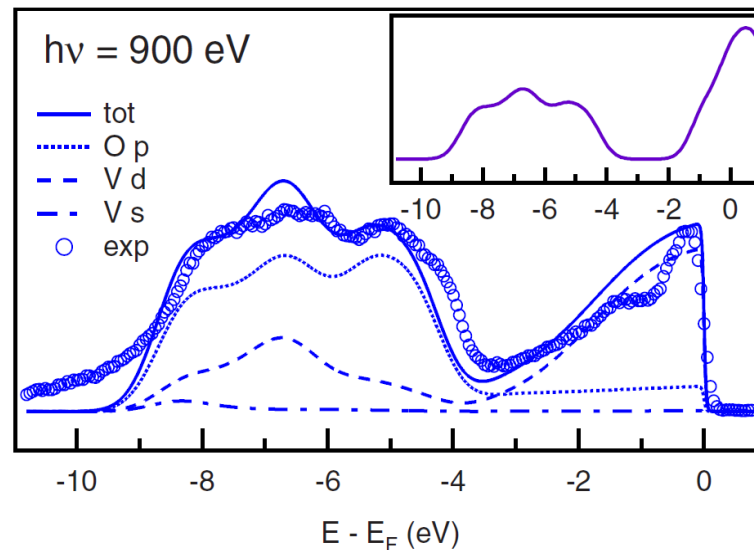
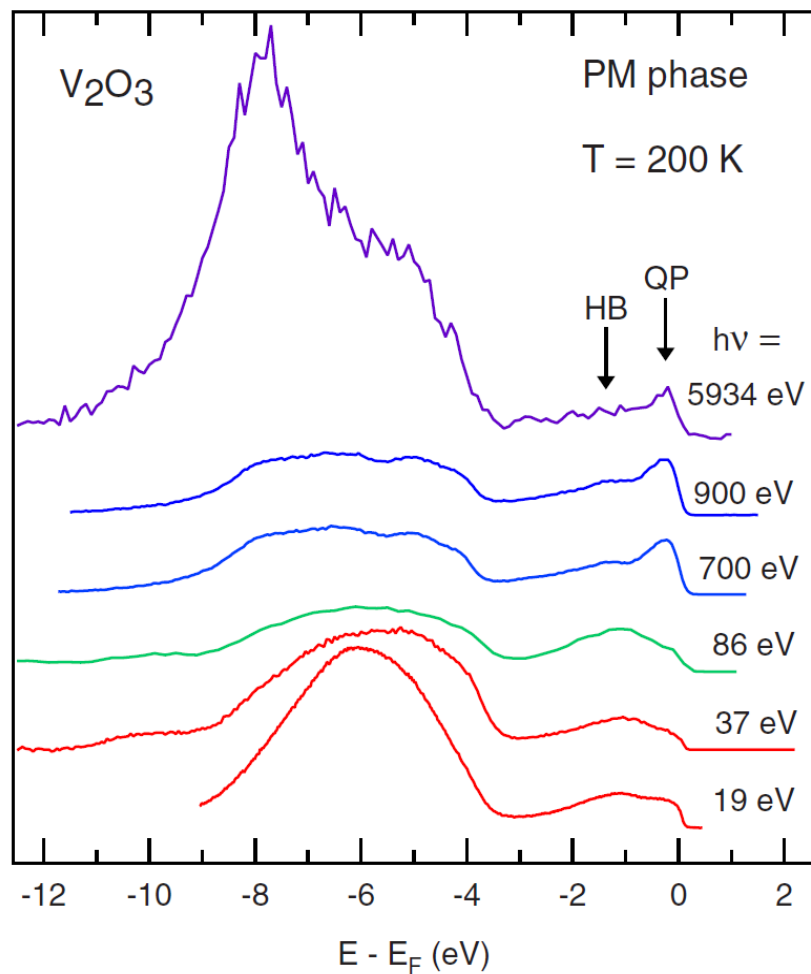
ZnO



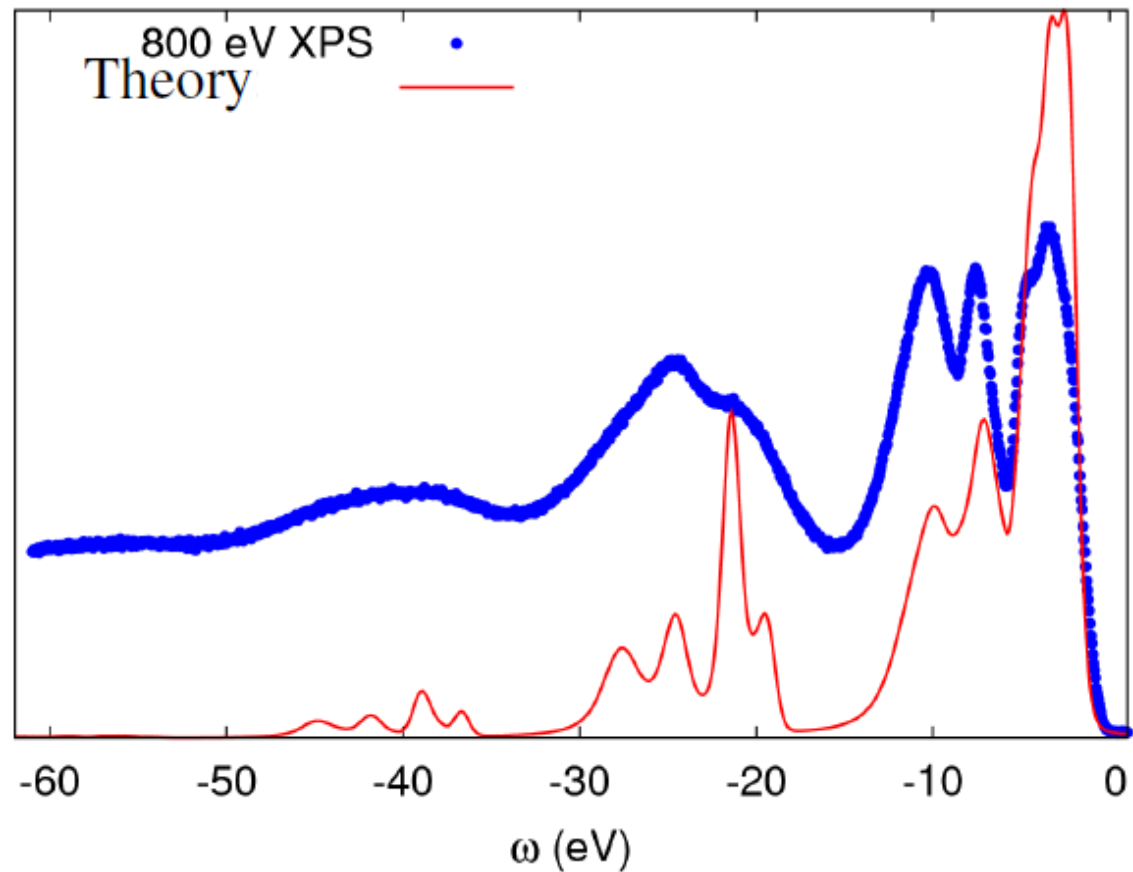
SnO₂



Matrix elements: V_2O_3

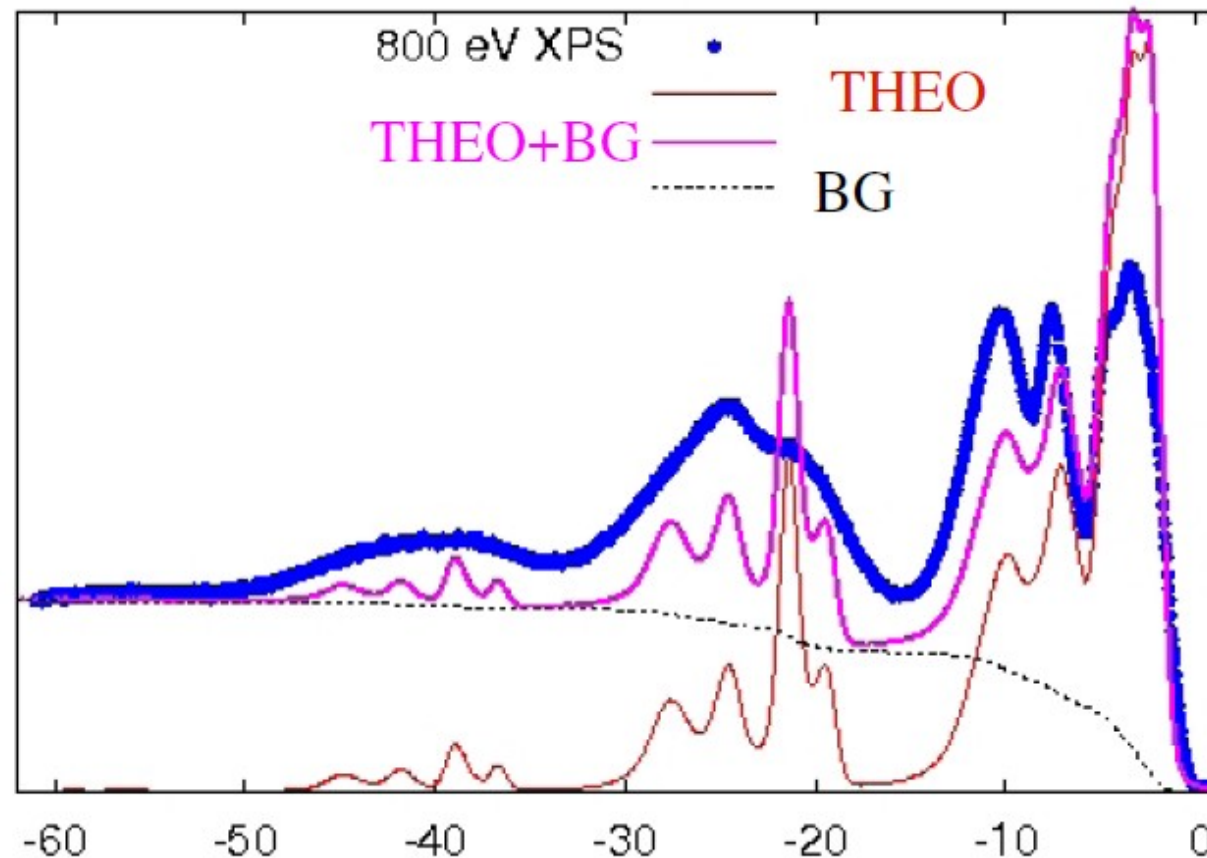


Getting closer to what is really measured



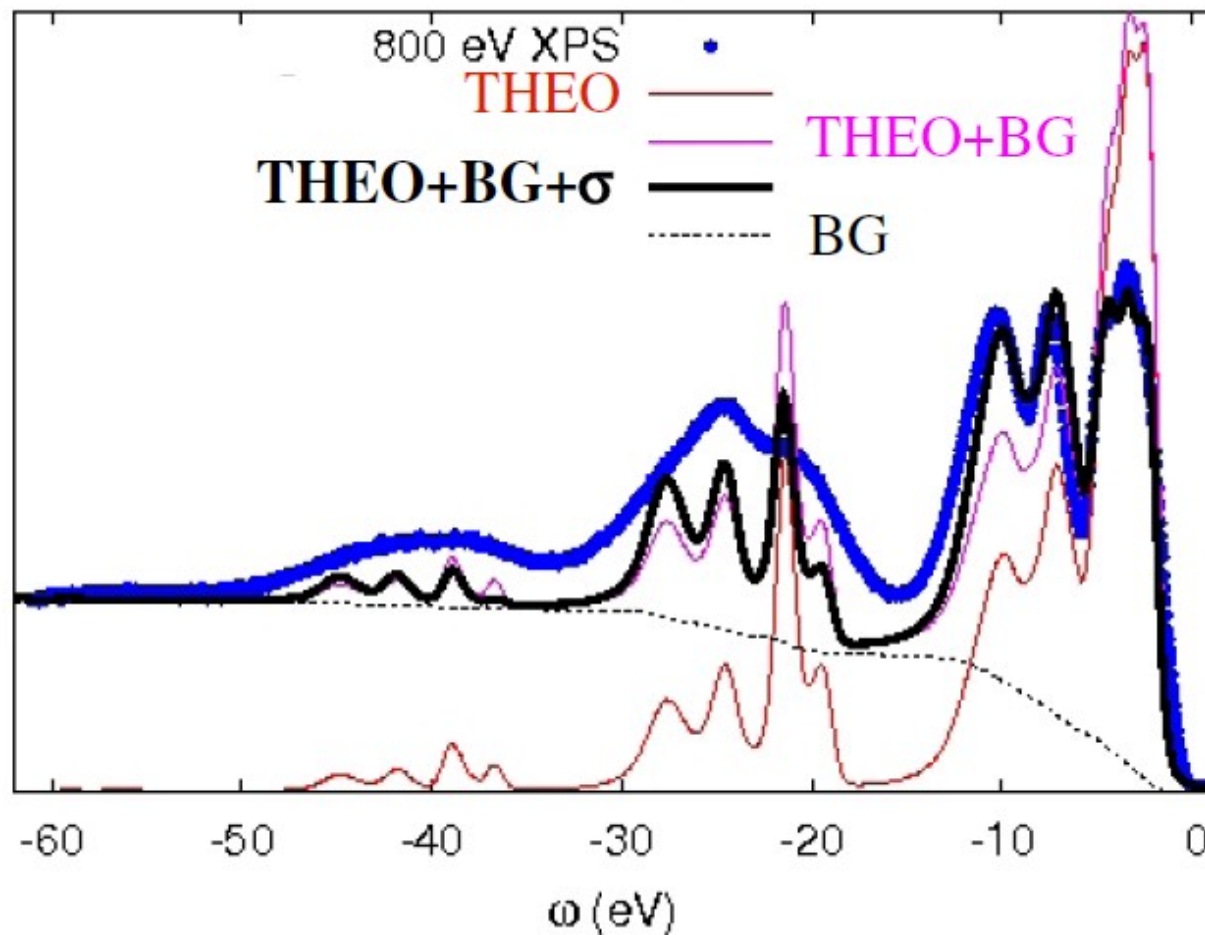
Spectral function

Getting closer to what is really measured



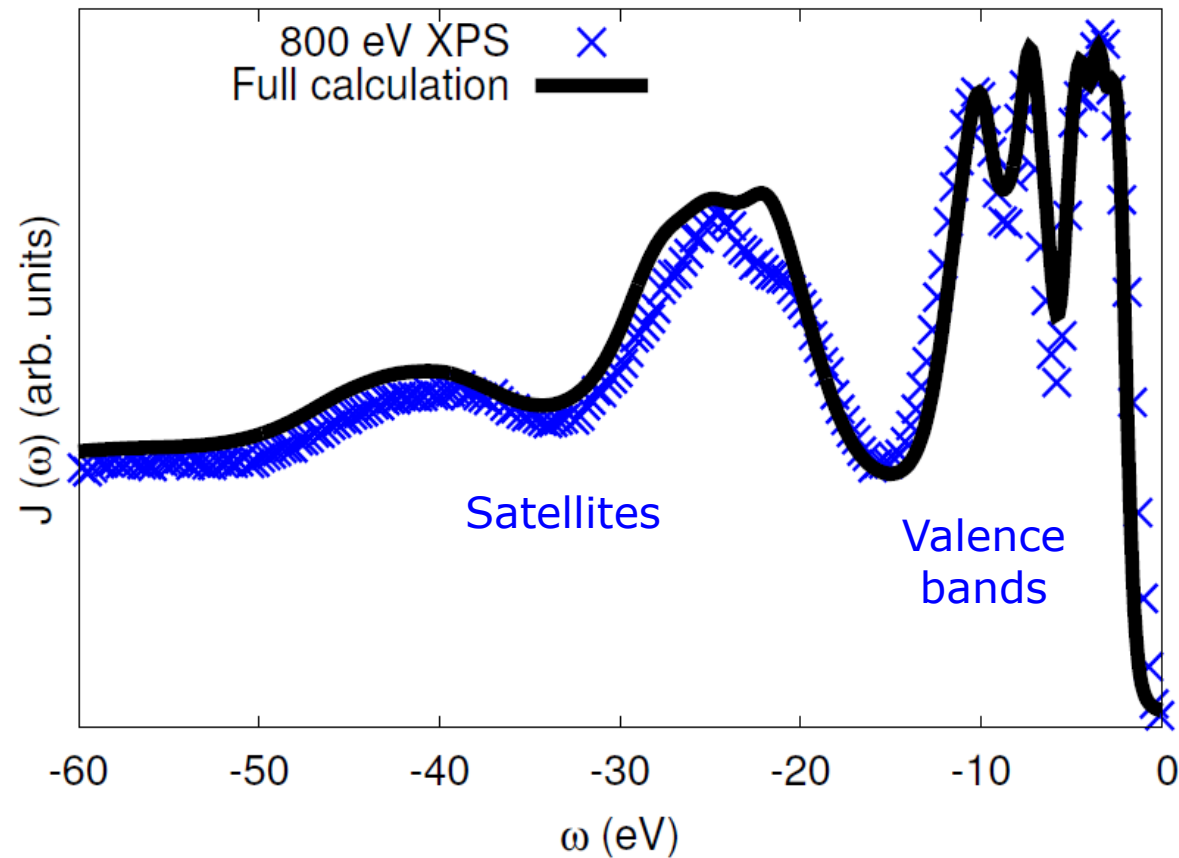
+ Background

Getting closer to what is really measured



+ Matrix elements

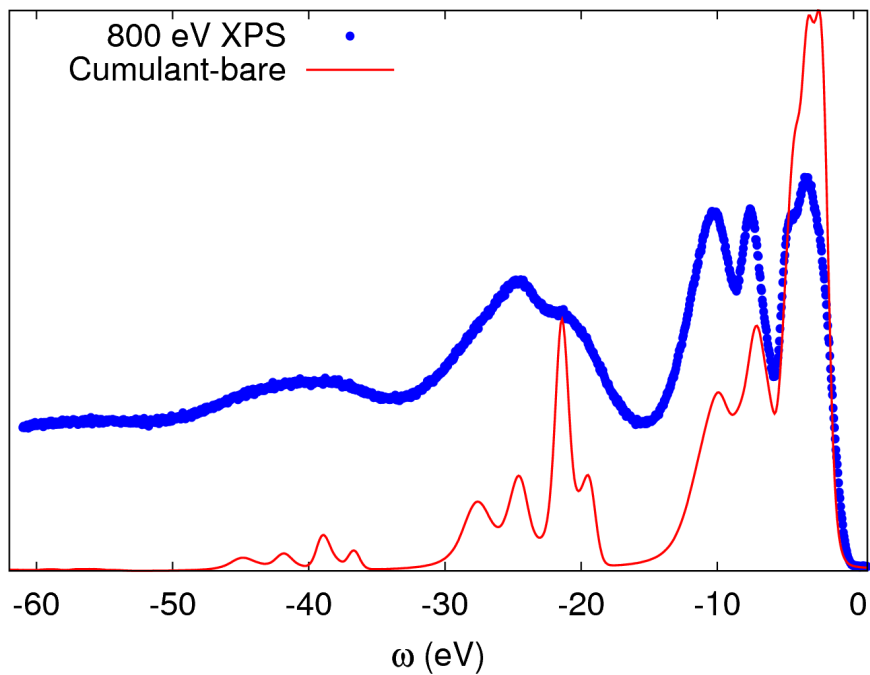
Getting closer to what is really measured



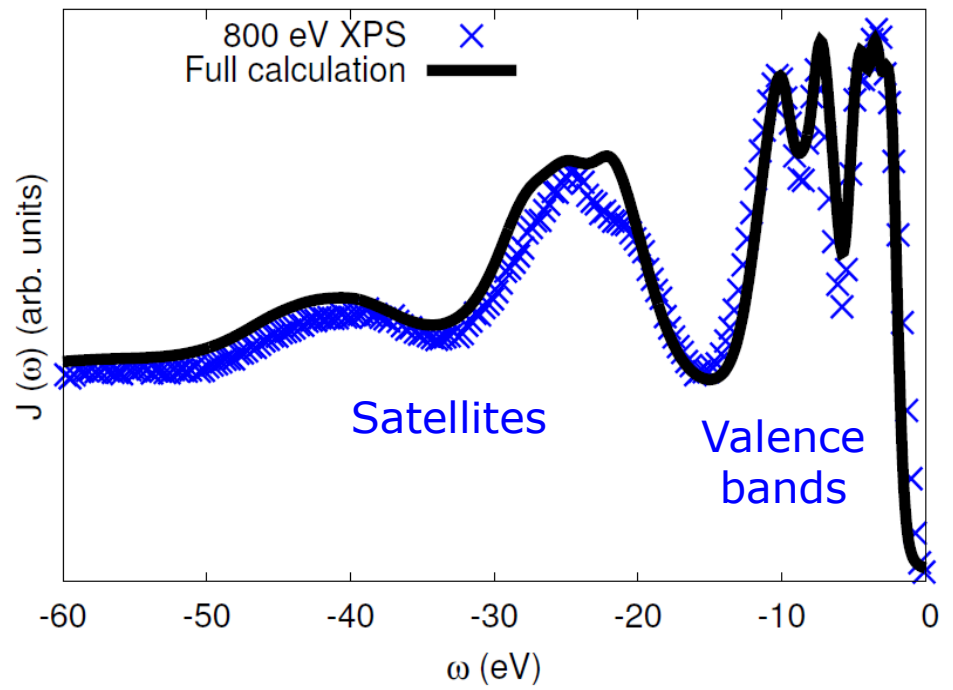
+ Interference/extrinsic effects

Getting closer to what is really measured

Spectral function

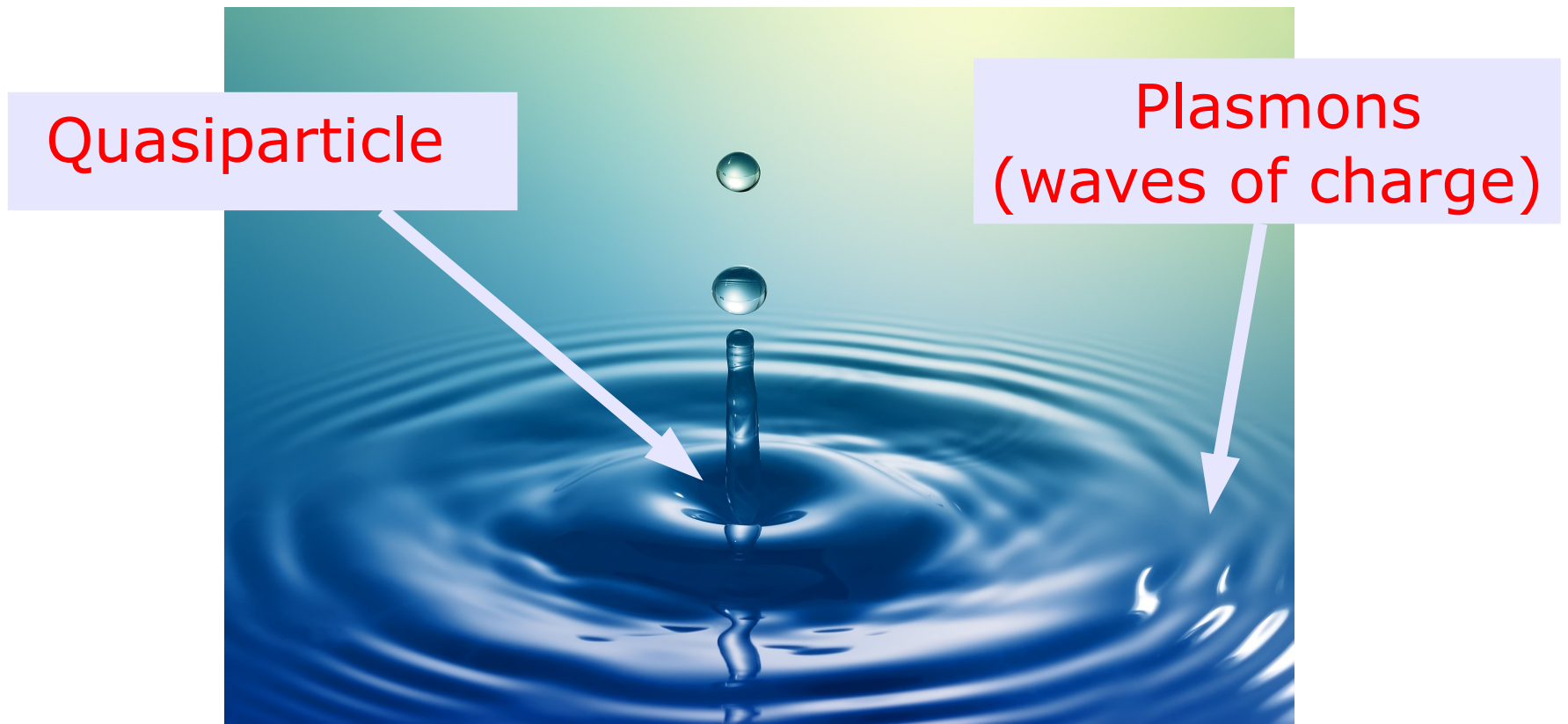


Photocurrent



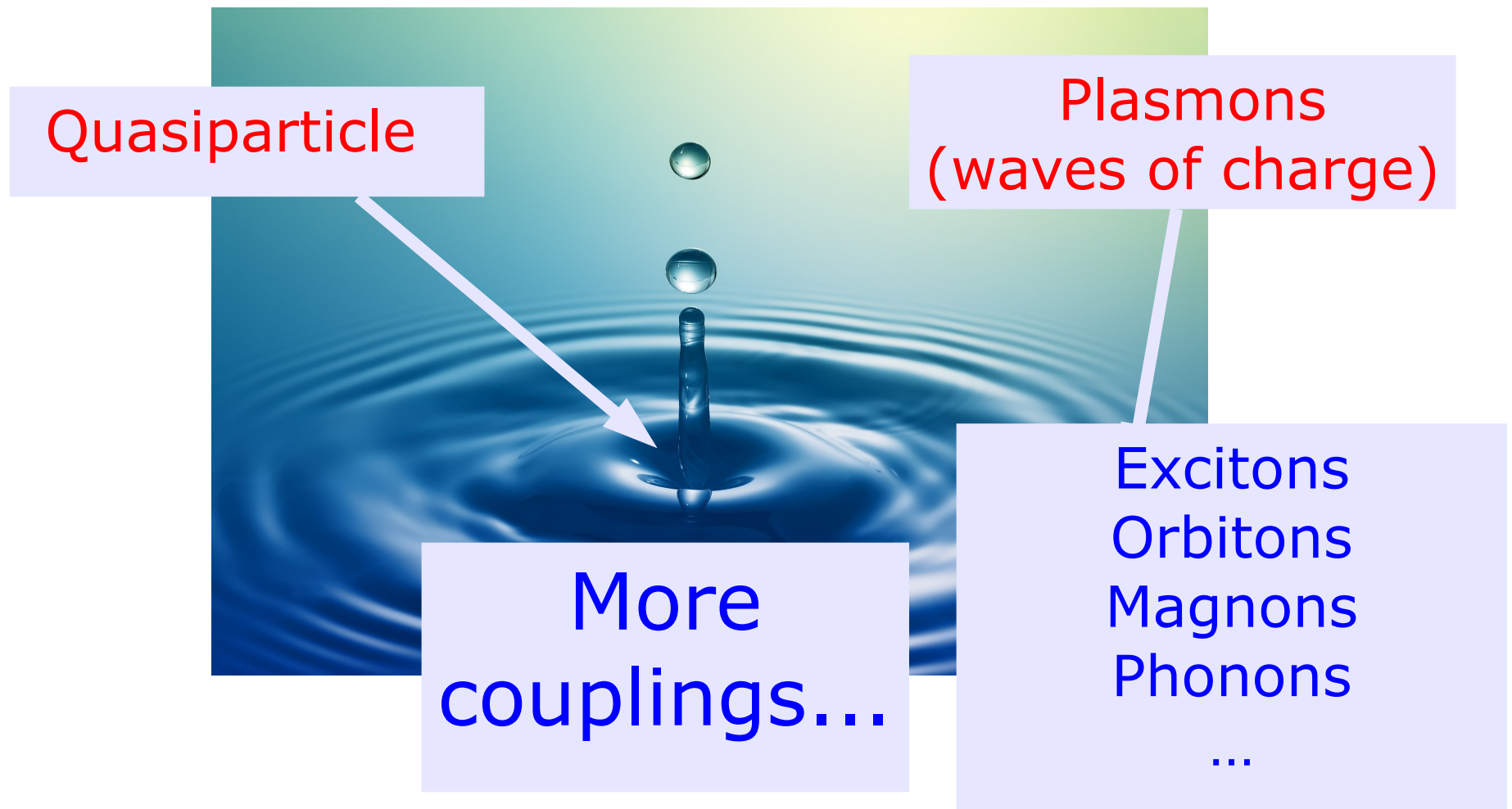
Correlation = coupling of excitations

Adding/removing electrons: reaction of all the others

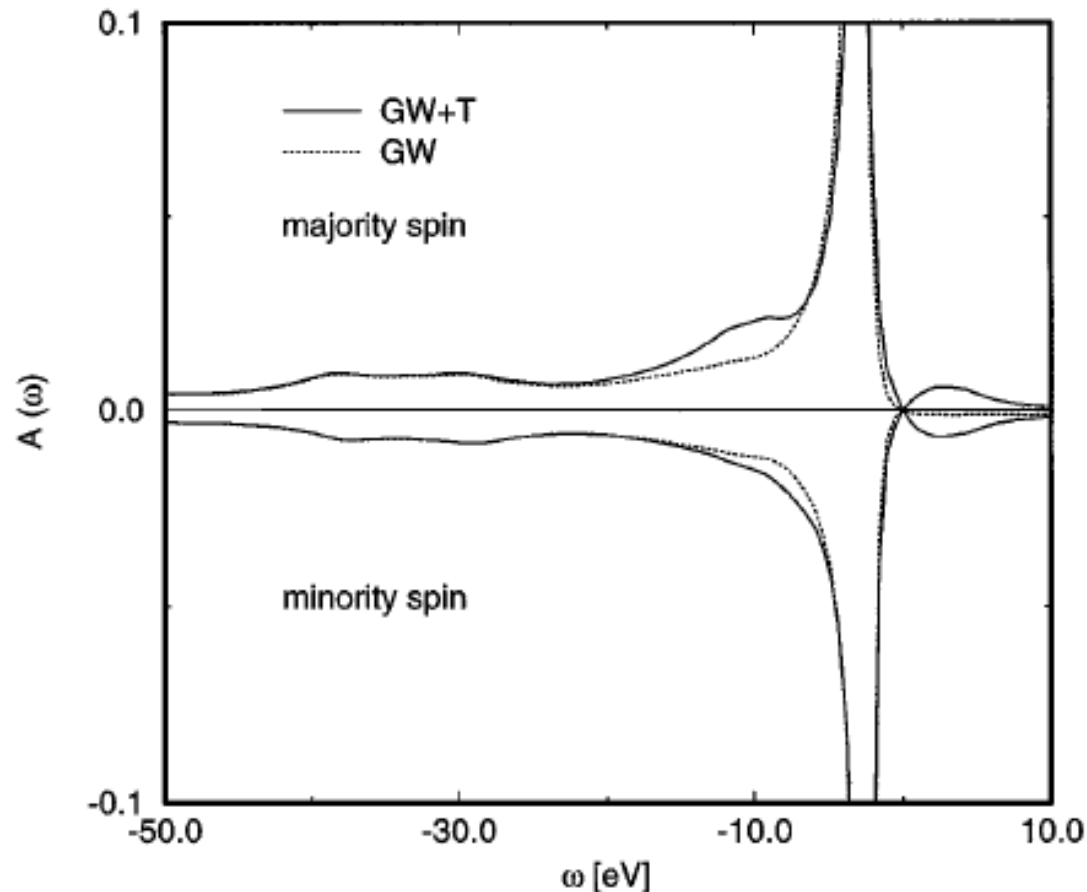


Correlation = coupling of excitations

Adding/removing electrons: reaction of all the others



Outlook: coupling with other excitations

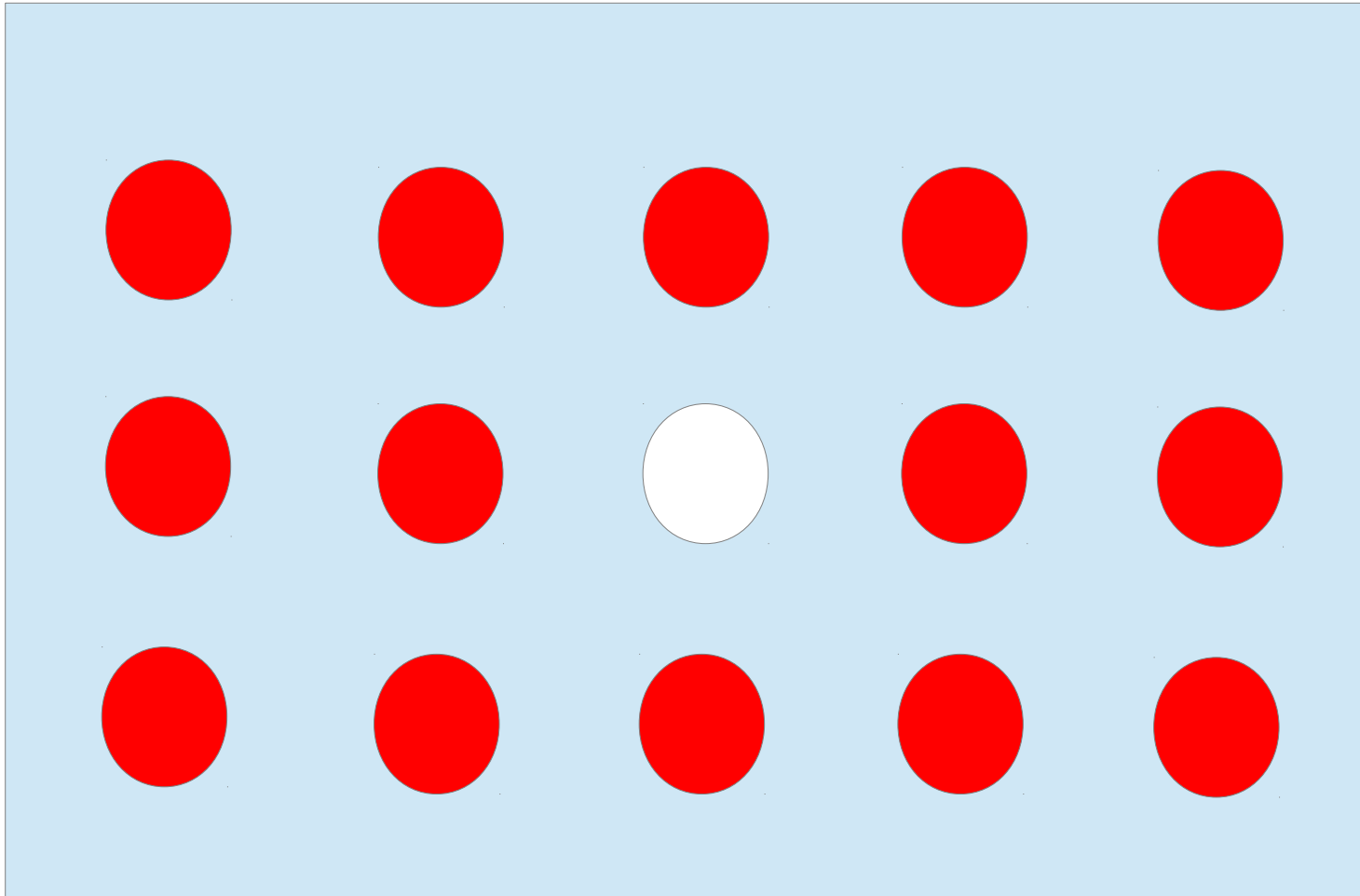


6 eV satellite in Ni: 2-hole bound state

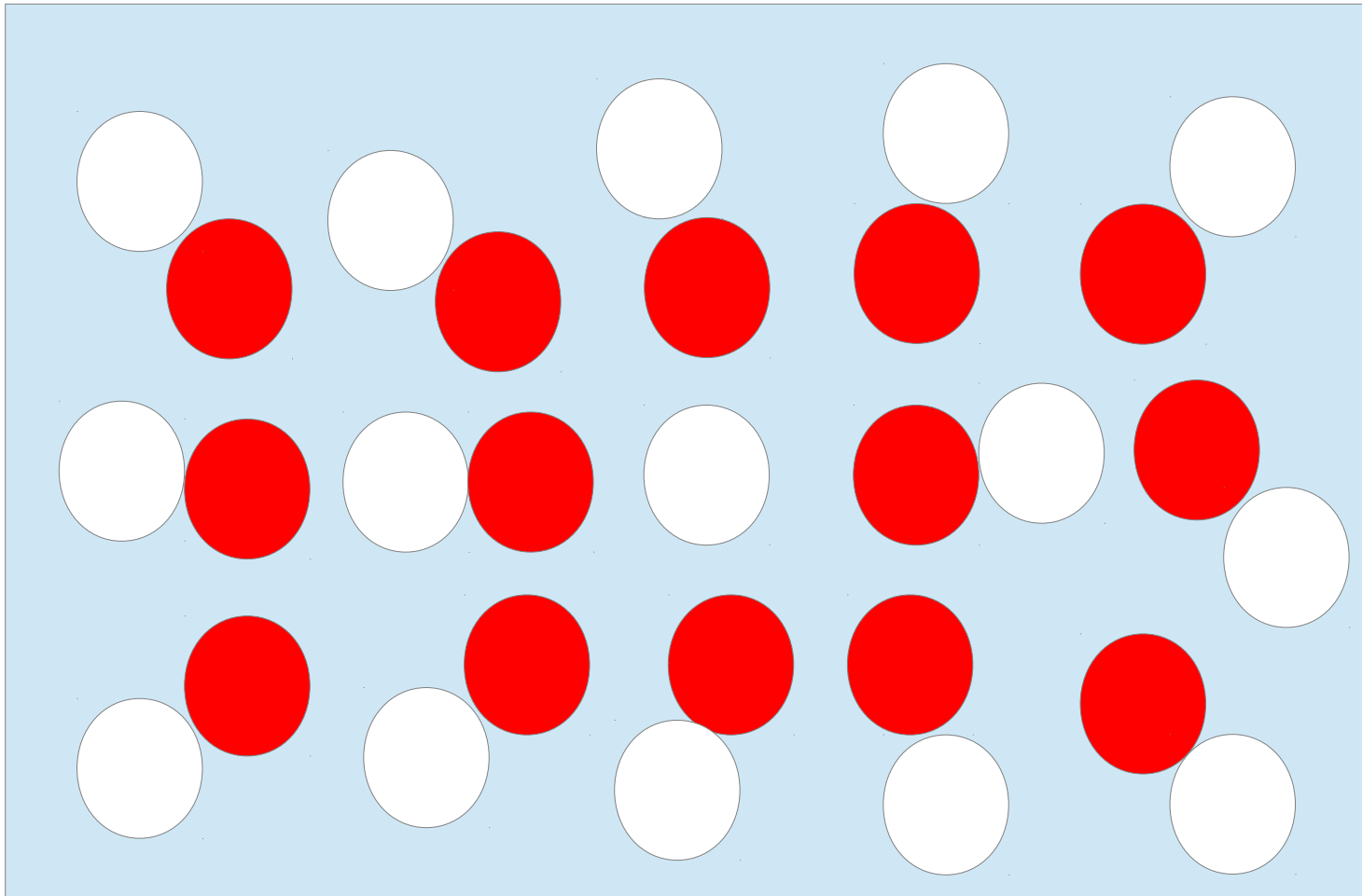
Springer, Aryasetiawan, Karlsson, PRL 80, 2389 (1998)

Back up slides

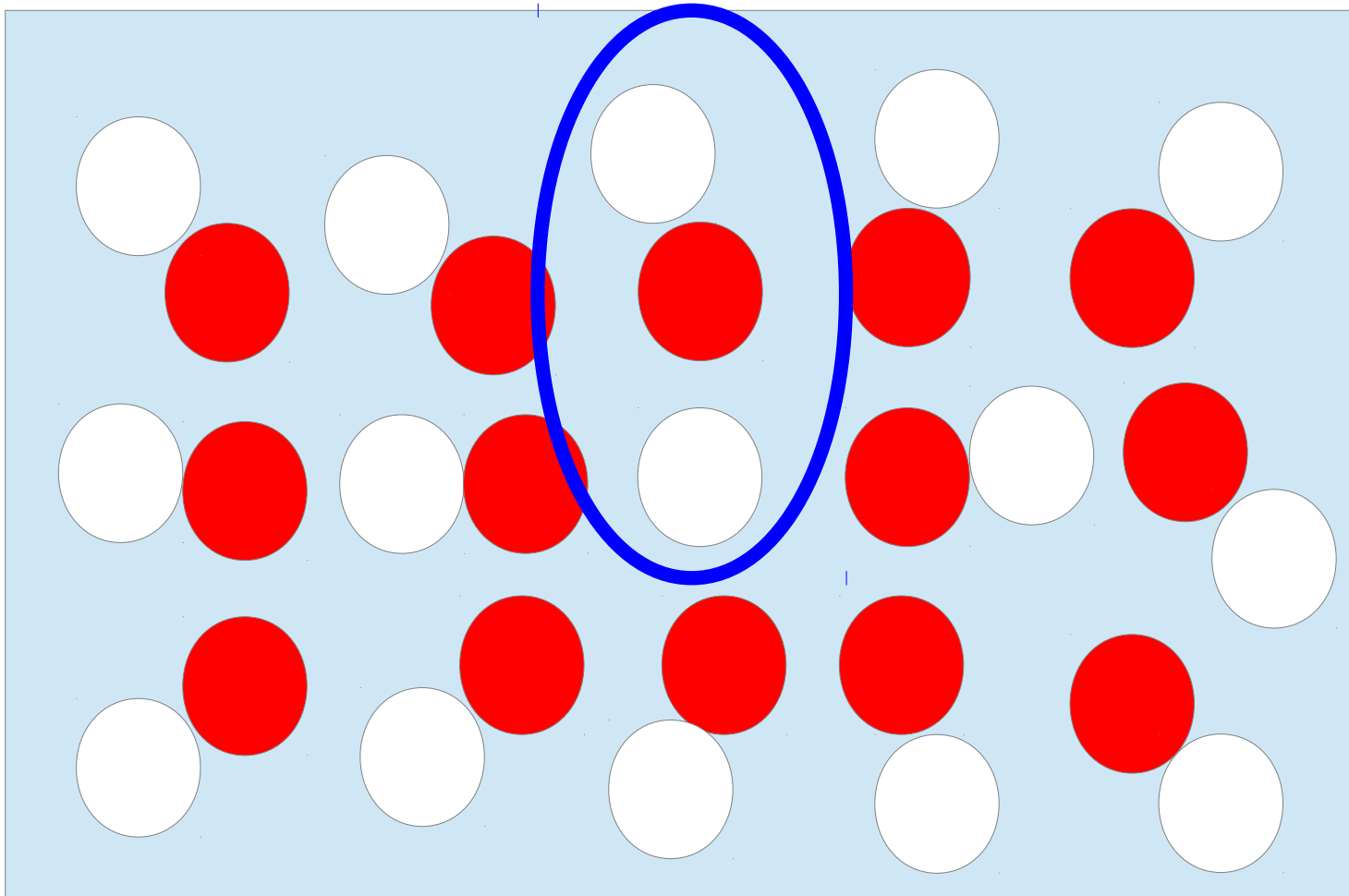
Interacting particles

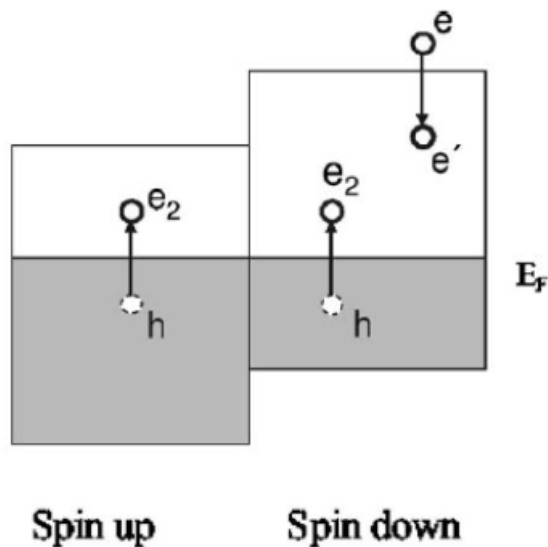


Interacting particles



Interacting particles





GW

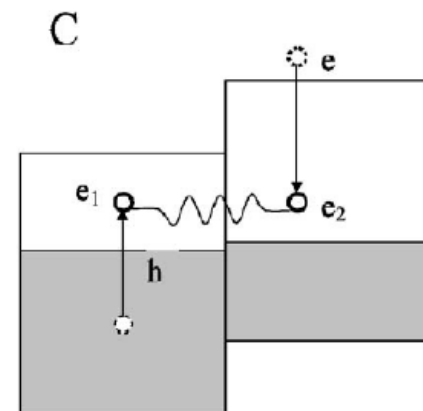
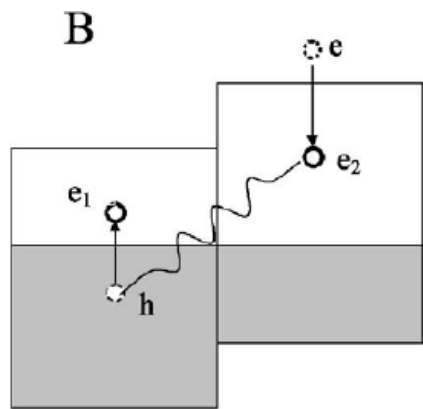
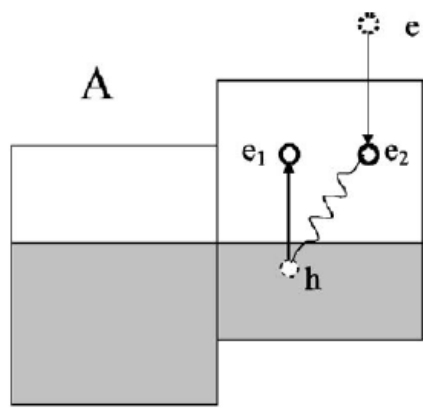
- Add one primary electron e , spin \downarrow
- Disexcitation $(e, \downarrow) \rightarrow (e', \downarrow)$
- Creation of electron-hole pairs e_2-h in both spin channels.

Note

- primary electron: final spin = initial spin (no spin flips)
- no interaction between primary electron and secondary particles
- analogously for additional hole

T matrix

- Add one primary electron e , spin \downarrow
- Disexcitation $(e, \downarrow) \rightarrow (e_2, \downarrow)$
- Creation of electron-hole pairs e_1-h in both spin channels
- Interaction between primary electron and hole of electron-hole pair (A,B)
- Interaction between primary electron and electron of electron-hole pair (C)



Spin up

Spin down

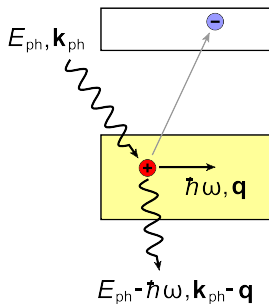
Note

- (B) spin flips: coupling with spin-waves, magnons, paramagnons
- analogously for additional hole

$$\mathcal{B}_f(\omega) = \int_{\omega}^{\mu} d\omega' f(\omega')$$

- Assumption: the bare signal produces a constant flux of secondary electrons at all energies
- The background is added to the theoretical spectrum (overall factor fitted to experiment)

Probing Neutral Electronic Excitations



Inelastic scattering of

Electrons

- electron-energy loss spectroscopy (EELS)
- low \mathbf{q}

X-rays

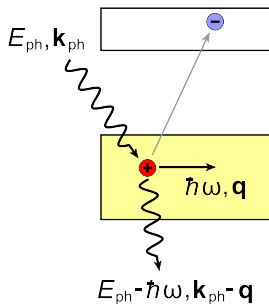
- non-resonant inelastic x-ray scattering (IXS)
- large \mathbf{q}

➔ charge density fluctuations (interband transitions, excitons, plasmons)

charge-density response function $\chi(\mathbf{r}_1, \mathbf{r}_2, t_1 - t_2)$

$$\chi(\mathbf{r}_1, \mathbf{r}_2, t_1 - t_2) = \frac{1}{i\hbar} \langle N | \mathcal{T} \Delta n(\mathbf{r}_1, t_1) \Delta n(\mathbf{r}_2, t_2) | N \rangle$$

Probing Neutral Electronic Excitations



Inelastic scattering of

Electrons

- electron-energy loss spectroscopy (EELS)
- low \mathbf{q}

X-rays

- non-resonant inelastic x-ray scattering (IXS)
- large \mathbf{q}

➔ charge density fluctuations (interband transitions, excitons, plasmons)

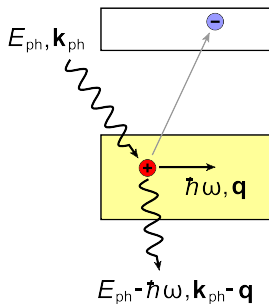
charge-density response function $\chi(\mathbf{r}_1, \mathbf{r}_2, t_1 - t_2)$

$$\chi(\mathbf{r}_1, \mathbf{r}_2, t_1 - t_2) = \frac{1}{i\hbar} \langle N | \mathcal{T} \Delta n(\mathbf{r}_1, t_1) \Delta n(\mathbf{r}_2, t_2) | N \rangle$$

loss function – $\text{Im} \varepsilon^{-1}(\mathbf{q}, \omega)$

$$\varepsilon^{-1}(\mathbf{q}, \omega) = 1 + v(\mathbf{q})\chi(\mathbf{q}\mathbf{q}, \omega)$$

Probing Neutral Electronic Excitations



Inelastic scattering of

Electrons

- electron-energy loss spectroscopy (EELS)
- low \mathbf{q}

X-rays

- non-resonant inelastic x-ray scattering (IXS)
- large \mathbf{q}

➔ charge density fluctuations (interband transitions, excitons, plasmons)

charge-density response function $\chi(\mathbf{r}_1, \mathbf{r}_2, t_1 - t_2)$

$$\chi(\mathbf{r}_1, \mathbf{r}_2, t_1 - t_2) = \frac{1}{i\hbar} \langle N | \mathcal{T} \Delta n(\mathbf{r}_1, t_1) \Delta n(\mathbf{r}_2, t_2) | N \rangle$$

loss function – $\text{Im} \epsilon^{-1}(\mathbf{q}, \omega)$

$$\epsilon^{-1}(\mathbf{q}, \omega) = 1 + v(\mathbf{q})\chi(\mathbf{q}\mathbf{q}, \omega)$$

dynamic structure factor $S(\mathbf{q}, \omega)$

$$S(\mathbf{q}, \omega) \sim \chi(\mathbf{q}\mathbf{q}, \omega)$$