



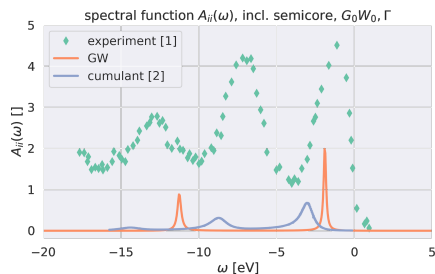
# Dynamical vertex corrections beyond GW from time-dependent density-functional theory

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# The problem with satellites ..

- Satellites in photoemission spectra are often (but not necessarily!) plasmon excitations
- Satellites are very sensitive to system details<sup>[2]</sup>
- GW has a satellite: → not the plasmon satellite!
- Plasmons were previously explained with the GW+C approach<sup>[3,4]</sup> → modification of G
- GW+C is limited to a coupling to plasmons
- Alternative: Use vertex corrections (avoid cumulant)

experimental photoemission spectrum of sodium<sup>[1]</sup>



Can we explain plasmon satellites by (dynamical) vertex corrections in  $W$ , instead of the GW+C approach?

[1] Höchst H. et al. *Z. Phys.B: Condens. Matter* 30 (1978), 145

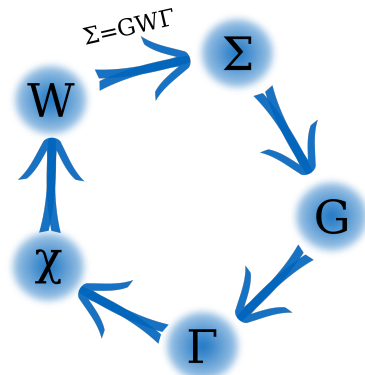
[2] Zhou J. et al. *Phys. Rev. B* 97(3) (2018), 035137

[3] Aryasetiawan F. et al. *Phys. Rev. Lett.* 77 (1996), 2268

[4] Guzzo M. et al. *Phys. Rev. Lett.* 107 (2011), 166401

# Vertex corrections $\Sigma = \text{GW}\Gamma \dots ?$

- In the common GW-approximation<sup>[1]</sup>:  $\Gamma = 1$
- $\Gamma$  is needed to calculate  $\Sigma$  ("outer") and  $\chi$  ("inner")
- In a qualitative picture:
  - The inner vertex describes the response of a the system as fermionic
  - The outer vertex describes the charge which feels the response as a fermion
- $\Gamma \rightarrow \frac{\delta\Sigma}{\delta G}$  Approximate as  $\frac{\delta v_{xc}(r,t)}{\delta n(r',t')} = f_{xc}(r, r'; t - t')$
- Common in the past: Adiabatic LDA
 
$$f_{xc}^{ALDA} \propto \delta(r - r')\delta(t - t')$$
- Commonly in  $\text{GW}\Gamma$  the  $f_{xc}$  is static<sup>[2-5]</sup>, but  $f_{xc}$  should be  $\omega$  dependent<sup>[6,9]</sup> and not completely local<sup>[7-9]</sup>



Can we include a  $q$ - and  $\omega$ -dependent  $f_{xc}$  from TDDFT<sup>[7]</sup> in the GW calculation?

[1] Hedin L. *Phys. Rev.* 139 (1965), A796

[2] Hybertsen M. S. et al. *Phys. Rev. B* 34 (1986), 5390

[3] Del Sole R. et al. *Phys. Rev. B* 49 (1994), 8024

[4] Morris A. et al. *Phys. Rev. B* 76 (2007), 155106

[5] Maggio E. et al. *J. Chem. Theory Comput.* 13(10) (2017), 4765

[6] Maebashi H. et al. *Phys. Rev. B* 84 (2011), 245134

[7] Reining L. et al. *Phys. Rev. Lett* 88 (2002), 066404

[8] Schmidt P. et al. *Phys. Rev. B* 96 (2017), 205206

[9] Panholzer M. et al. *Phys. Rev. Lett.* 120 (2018), 166402

# Effect of the inner vertex studied by a tabulated $f_{XC}$ -kernel [1]

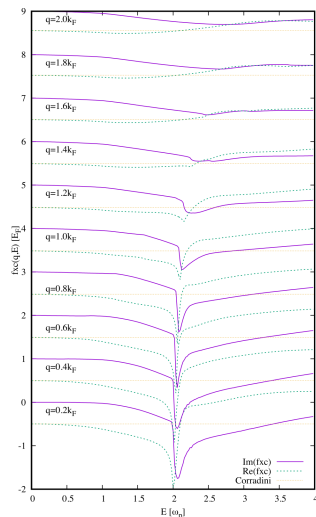
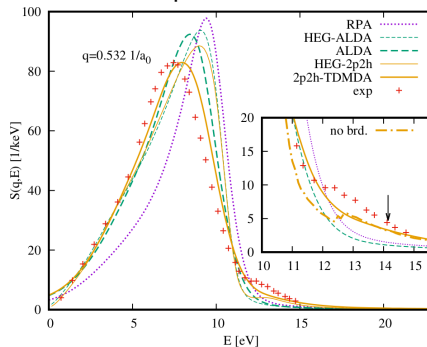


FIG. 1: The 2p2h kernel as a function of energy at  $r_s = 4$  and at momenta starting from  $q = 0.2k_F$  up to  $q = 2k_F$  in steps of  $\Delta q = 0.2k_F$  is shown. With increasing momentum the lines are offset in y-direction by  $1E_F$ .

## Plasmon and double plasmon in IXS of Na $\propto \text{Im}[\chi]$



- correlated EOM with 1 & 2 particle excitations [2]
- TDDFT  $f_{XC}$  of the HEG (tabulated [3]): "2p2h-kernel"
- dominant feature of the kernel is a pole at  $2\omega_p$
- Using this  $f_{XC}$ : plasmon and double-plasmon in  $S(q, \omega)$  [2]
- $f_{XC}$  must be  $\omega$ -dependent to describe the double plasmon

[1] Panholzer M. et al., *Phys. Rev. Lett.* 120 (2018), 166402

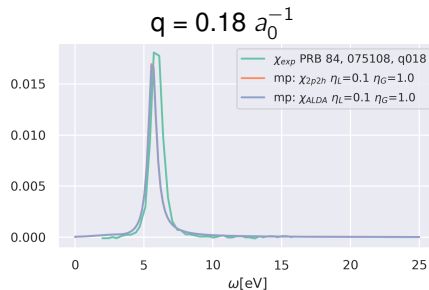
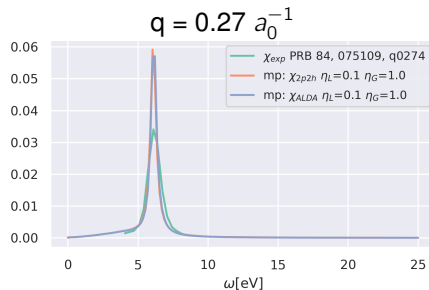
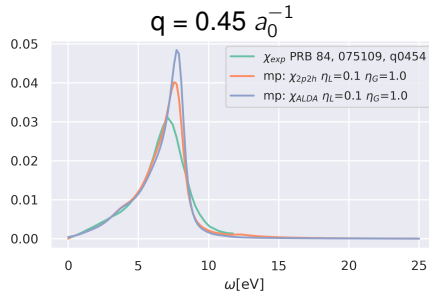
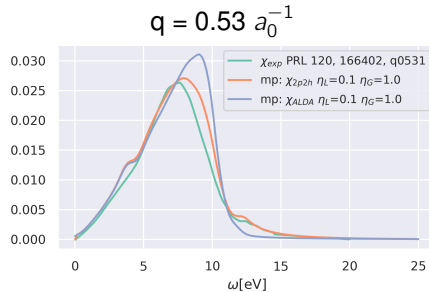
[2] Böhm H. M. et al. *Phys. Rev. B* 82, (2010), 224505

[3] <https://etsf.polytechnique.fr/research/connector/2p2h-kernel>

⇒ This  $f_{XC}$  kernel is both q- and  $\omega$ -dependent!

# Can we reproduce $\chi_{2p2h}$ with a multipole model?

$$\chi_{2p2h} = \frac{\chi_0}{1 - v\chi_0 - f_{xc}\chi_0}$$



- model reproduces  $\chi_{2p2h} \rightarrow$  systematically better than ALDA compared to experiment
- effect of  $f_{xc}$  in  $\chi$  is weak for small values of  $q \Rightarrow \chi_0$  is important!

In the small  $q$ , the  $f_{xc}$ -effect is small  $\Rightarrow$  Maybe in the outer vertex this is different?

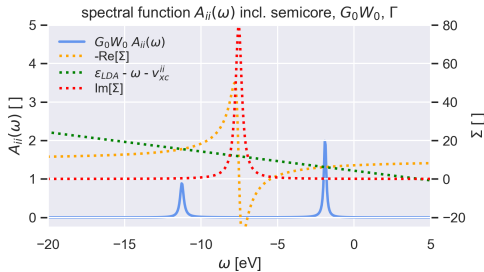
# The outer vertex

- Add analytical term of the same structure as the 2p2h-kernel

$$\tilde{W}(\omega') = [1 + (\nu + \alpha f_{xc}(\omega'))]\chi(\omega')\nu \quad \Rightarrow \quad \Sigma = GW\Gamma = G\tilde{W}$$

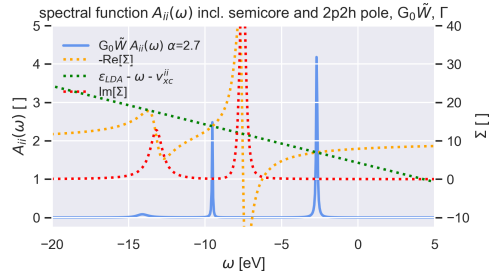
$\alpha$  is not a free parameter  $\rightarrow$  determined from  $f_{xc}(\omega = 0)$

without kernel



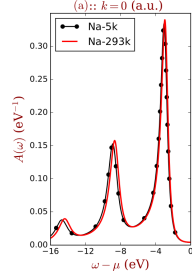
- $\rightarrow$  satellite peak  $\neq \omega_p (\simeq 6\text{eV})$
- $\rightarrow$  no  $2 * \omega_p$  satellite

including kernel



- $\rightarrow$  satellite shifted towards  $\omega_p$
- $\rightarrow$  quasiparticle distance decreased
- $\rightarrow 2 * \omega_p$  satellite appears

cumulant<sup>[1]</sup>

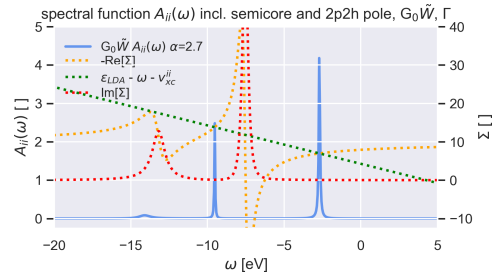
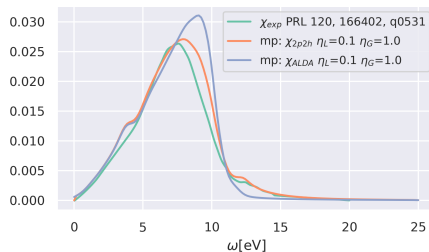


All seen effects are caused purely because of the  $\omega$ -dependence of  $f_{xc}$ !

[1] Zhou J. et al. *Phys. Rev. B* 97(3) (2018), 035137

# Conclusion

- The inner model vertex introduces  $2\omega_p$  in  $\tilde{\chi}$ 
  - The effect of the inner vertex is small for small  $q$
- The outer model vertex predicts qualitatively correct spectral function
  - The outer vertex shifts the plasmon satellite and creates the  $2\omega_p$
- Vertex corrections with  $f_{xc}(\omega)$  are a different way to get the same result as the cumulant but open a possible route towards coupling to other bosons
- Other materials are not so close to the HEG
  - if needed, a connector can be employed<sup>[1–4]</sup>



- [1] Ayoub Aouina, *MSc Thesis (2019)*, Development of functionals for observables  
 [2] Marco Vanzini, *PhD Thesis (2018)*, Auxiliary systems for observables: dynamical local connector approximation for electron addition and removal spectra

- [3] Panholzer M. et al., *Phys. Rev. Lett.* 120 (2018), 166402  
 [4] Vanzini M. et al., *arXiv:cond-mat.other (2019)*, 1903.07930



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**Thank you for your attention!**